W O R L D   M E T E O R O L O G I C A L   O R G A N I Z A T I O N

WEATHER, CLIMATE AND WATER

CHy Project: Assessment of the Performance of Flow Measurement Instruments and Techniques

DISCHARGE UNCERTAINTY EXAMPLE: WEIGHING AND TIMING MEASUREMENTS OF DISCHARGE

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1 March 2009

3. Discharge Uncertainty Example: Weighing and Timing Measurements of Discharge

3.1 Introduction

Discharge measurements, like all measurements, are an approximation of the “true” values. The quantification of uncertainty is needed to give a complete measurement result (Taylor and Kuyatt, 1994). The uncertainty of a discharge measurement is dependent on the instrumentation, method, environmental conditions, and care with which the measurement is made.

Several different types of instrumentation and techniques can be used to measure discharge. Depending on the instrumentation, measurement techniques, and flow conditions, the computation of discharge and its associated uncertainty can be relatively simple or complex. The measurement of discharge by timing of a flow into a container and weighing the flow captured by the container is a relatively simple method of measuring a laboratory discharge. For this simple discharge measurement method, the estimation of the measurement uncertainty will be presented for three measured discharges, demonstrating the application of methods recommended by GUM for computing uncertainty.

The example demonstrates that uncertainty can be computed using a combination of measurement data and information from instrumentation specifications and laboratory conditions. It also illustrates that repeated measurements (increased sampling) can be used to reduce the uncertainty of the final (or averaged) measurement.

3.2 Estimating Measurement Uncertainty

As presented in previous sections, GUM classifies standard uncertainties either as type A, computed from measured data, or type B, estimated by methods not using measured data. Generally, random effects are captured by a type A uncertainty and systematic effects are captured by a type B uncertainty. Exceptions to the generalization can occur, and random effects may be estimated using a type B uncertainty, while systematic effects may be estimated from the statistical analysis of measurement data (Type A). Depending on the availability of measurement data, an uncertainty estimate might consist of only type B uncertainties.

Sources of uncertainty can change from being either random or systematic, depending on the measurement process. For example, if a group of discharge measurements is made by an individual technician, the technician contributes a systematic uncertainty to the discharge measurements, but if the group of discharge measurements is made by many different technicians, the technicians contribute a random uncertainty to the discharge measurements.

Standard uncertainties assume that uncertainties can be “modeled by probability distributions quantified by variances and standard deviations” (UKAS, 2007). Regardless of the source of uncertainty, the standard deviation is defined as the standard uncertainty. If data is available from which a standard deviation can be computed (type A uncertainty), a normal distribution is typically assumed. If other sources of information are used to determine the standard uncertainty (type B uncertainty), other probability distributions may be used. This may require additional processing to convert the initial information for type B uncertainties into a form that is equivalent to one standard deviation for the population. The processing required is dependent on the type of probability distribution that is assumed such as, triangular, rectangular or normal.

The combined standard uncertainty, u_c, is computed from the individual standard uncertainties using equation 2.7 (sec 2) or equation 2.10 (sec 2). Similar to a standard deviation, it covers approximately 68% of the possible measurement outcomes. The expanded uncertainty, U, computed

by equation 2.12 (sec 2) typically covers 95% of the possible measurement outcomes and is the combined standard uncertainty multiplied by a coverage factor that is a function of sample size.
Applying the GUM to an UA without any other guidance can be a daunting task, especially for novices to UA. Several national standardization and accreditation agencies have written succinct guides to assist in the application of the GUM to determining measurement uncertainty. Two such guides that novices to UA will find helpful are: "The Expression of Uncertainty and Confidence in Measurement" (UKAS, 2007) and "Guidelines for Evaluation and Expressing the Uncertainty of NIST Measurement Results" (NIST, 1994).

3.3 Measurement of Discharge by Weighing and Timing

The measurement of discharge by weighing and timing a constant flow is often used to calibrate discharge measuring devices, such as small flumes, in hydraulic laboratories. The measurement of discharge by timing and weighing a captured amount of water is computed from:

$$Q = \frac{(w_2 - w_1)}{\gamma t} \tag{3.1}$$

where Q is the discharge in volume per unit time, w is the weight of the container and water, t is the elapsed time, and γ is the specific weight of water. The subscripts 1 and 2 indicate the initial and final weight measurement, respectively. The specific weight of water can be computed from $\gamma = \rho g$ where g is the acceleration due to gravity and ρ is the density of water. Because weight is defined as a force in the technical setting of the hydraulic laboratory (Thompson and Taylor, 2008), specific weight is defined as force per volume.

The uncertainty of a discharge measurement by weighing and timing is dependent on the data reduction equation, what instrumentation is used and how the measurement is made. Equation 3.1 is the data reduction equation. From this equation, sensitivity factors for uncertainty sources can be computed. Information on the accuracy and resolution of the instrumentation used to make the measurements, the values used for the acceleration due to gravity and water density and the process used in the laboratory to make the measurement are necessary to compute the uncertainty of the measurement.

3.3.1 Laboratory Measurement Process

The discharge is measured in an indoor hydraulic laboratory at the exit of the small flume test stand. The small flume test stand allows the flow exiting from the flume to be diverted by a laboratory technician into a large drum. The measurement process starts with the empty drum being weighed by laboratory staff to determine the initial weight. The initial weight is recorded by hand in a log book. The flow is then diverted into the drum and timed manually by laboratory staff until the flow is diverted away from the drum. The drum is again weighed by laboratory staff to determine the final weight. The elapsed time and final drum weight are recorded in the log book. A valve to the drum is then opened to allow the drum to empty. The data recorded for the measurement are the initial weight, final weight and elapsed time. The discharge is computed using equation 3.1 and assuming γ = 9.8067 kN/m^3 (62.43 lb/ft$^{3)}$.

A hand-held stop watch is used to time the filling of the drum. An electronic floor scale is used to weigh the drum and water. The flow through the flume is supplied by a pump system that uses a constant head tank. The discharge measurements are made in a laboratory that is heated in the winter but is not air conditioned. Air temperatures in the laboratory range annually from about 15 to 40 ^{0}C and water temperature at the time of the experiment is not measured.

3.3.2 Uncertainty Sources

Sources of uncertainty for the discharge measurement include the accuracy and resolution of the floor scale, the accuracy and resolution of the stop watch, the laboratory staff, the steadiness of the flow, and the accuracy of the values used for the acceleration of gravity and water density. Temperatures of the water and the air are not measured during the discharge measurement and may affect the uncertainty of the instruments used and the discharge measured.

3.3.2.1 Time and weight

A stop watch is used to measure the time, *t*, in seconds to fill the drum. It has a manufacturer's accuracy of ± 5 s/d and a resolution of 0.01 s and an operating temperature range of 1 to 59 ^{0}C.

An industrial floor scale with a digital readout is used to weigh the empty and filled drum and has a manufacturer's accuracy of ± 0.1 kg (± 0.2 lbs) and a display resolution of 0.1 kg (0.2 lbs). The scale's digital display shows a rounded representation of the analog measurement resulting in a measurement resolution of one-half of the digits displayed (display resolution), 0.5 X 0.1kg = 0.05 kg. The scale has an operating temperature range of -10 to 40 ^{0}C. As the same scale is used to make both weight measurements, the uncertainties for w_1 and w_2 are correlated. Table 3.1 summarizes the available manufacturer information on the stop watch and floor scale used in the discharge measurement.

Table 3.1 Information from manufacturer specifications for uncertainty analysis.

Instrument	**Operating temperature range ^{0}C**	**Range**	**Accuracy**	**Display Resolution**
Floor Scale	-10 to 40	226.8 kg	0.1 kg	0.1 kg
Stop Watch	1 to 59	9hrs 59m 59.99s	0.0058%	0.01 s

3.3.2.2 Specific weight

The specific weight of water used to compute water volume from weight, γ = 9.8067 kN/m^3 (62.43 lb/ft^3) is based on the standard acceleration of gravity of 9.80665 m/s^2 (32.174 ft/s^2) and the density of water at 4 ^{0}C. Local gravity is related to latitude and can be estimated from the NOAA surface gravity prediction map (www.ngs.noaa.gov/cgi). The hydraulic laboratory is 25 ft above mean sea level and is located at latitude 30^0 20' 06" and longitude 89^0 30' 32". Using the NGS, NOAA calculator, local acceleration of gravity is estimated as 9.79336 m/s^2 (32.13045 ft/s^2); about +0.136% difference with the standard value. The laboratory water supply is pumped from an outdoor constant-head tank. Water temperature measurements are not made during the discharge measurement. It is estimated that the laboratory water temperature could range from 10 to 30 ^{0}C over a year. The uncertainty on the volume of water measured due to not correcting for temperature effects on water density range from +0.023% to +0.431%. This results in under measuring the volumetric discharge. The estimated mean value of the specific weight during discharge measurements is 9.7712 kN/m^3 (62.202475 lb/ft^3)and varies from 9.7911 kN/m^3 to 9.7514 kN/m^3 (62.329 to 62.076 lb/ft^3) The uncertainty due to using a specific weight of water of 9.8067 kN/m^3 (62.43 lb/ft^3) is summarized in table 3.2.

Table 3.2 Percent difference in specific weight of water from using standard gravity and water density at 4 °C.

°C	Specific Weight, standard gravity (kN/m³)	Percent difference due to temperature	Specific weight, local gravity (kN/m³)	Percent difference with standard value
4	9.8067 (std value)	0.003	9.7934	0.139
10	9.8044	0.026	9.7911	0.162
20	9.7918	0.154	9.7786	0.290
30	9.7646	0.434	9.7514	0.570

3.3.2.3 Repeatability of discharge measurements

Data from previous discharge measurements are available and can be used to estimate the standard uncertainty associated with the repeatability of the discharge measurement. Repeatability of the measurement is dependent on the steadiness of the flow and the skill of the laboratory staff operating the stop watch and diverting the flow. Standard deviations computed from prior experiments with replicated discharge measurements are plotted with mean flow rate in figure 3.1. Figure 3.1 contains discharge measurements from 109 discharge measurements that were obtained for 11 flume calibrations. The measured discharges range from 2.832 x 10^{-4} m^3/s to 15.574 x 10^{-4} m^3/s (0.01 to 0.55 ft^3/s).

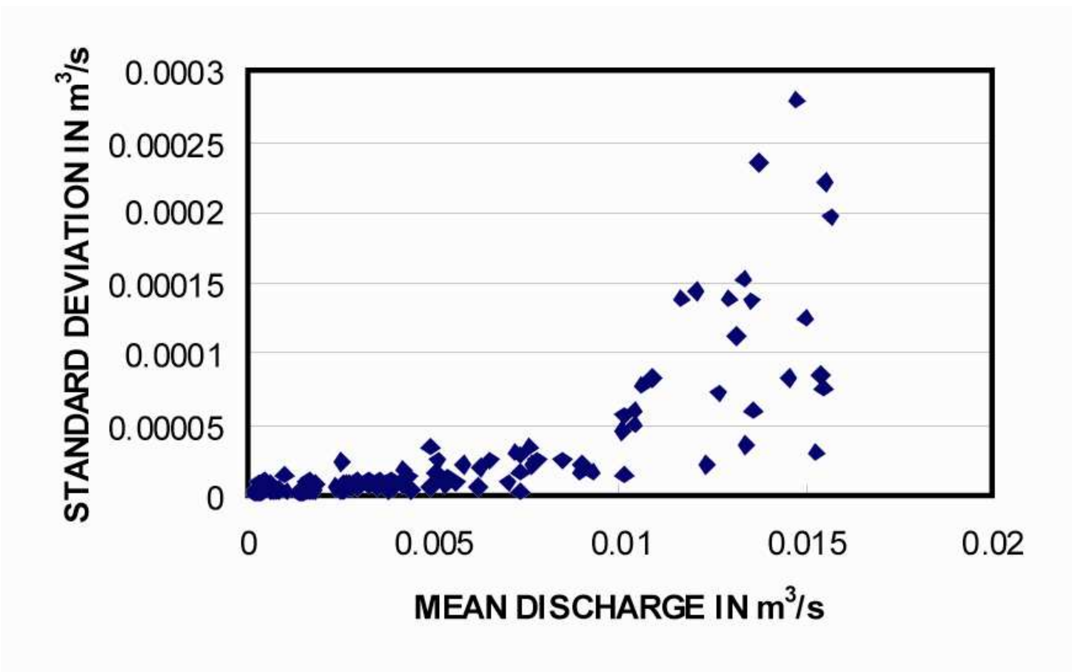

Figure 3.1 Standard deviations from previously repeated measurements plotted versus mean discharge.

3.3.3 Sensitivity Coefficients

Sensitivity coefficients, c_i, for each input in equation 3.1 can be computed by taking partial derivation with respect to each input.

$$\frac{\partial Q}{\partial w_2} = c_{w2} = \frac{1}{\rho g t} \tag{3.2}$$

$$\frac{\partial Q}{\partial w_1} = c_{w1} = \frac{-1}{\rho g t} \tag{3.3}$$

$$\frac{\partial Q}{\partial t} = c_t = \frac{-\left(w_2 - w_1\right)}{\rho g t^2} \tag{3.4}$$

$$\frac{\partial Q}{\partial \gamma} = c_\gamma = \frac{-\left(w_2 - w_1\right)}{\gamma^2 t} \tag{3.5}$$

Each standard uncertainty is multiplied by the appropriate sensitivity coefficient. This ensures that of all the standard uncertainties will be in a common unit. For this example the common unit is discharge in m^3/s.

3.4 Estimation of Uncertainty for Various Discharges

The estimated combined uncertainty is a function of the discharge and the duration of the test as well as the standard uncertainties of the input variables in equation 3.1. From previous measurements, it is also known that the flume discharge measurements have significant variability at a fixed flow rate that results from unintentional operating errors splashing water out of the drum when diverting the flow and timing the fill of the drum. Using equation 2.10 from section 2.2.2.4 with the input variables from the data reduction equation, 3.1, and the uncertainty from the measurement repeatability that captures the unintentional operator errors, the estimated combined uncertainty equation is:

$$u_c^2(y) = c_{w2}^2 u^2(w2) + c_{w1}^2 u^2(w1) + c_t^2 u^2(t) + c_\gamma^2 u^2(\gamma) + 2c_{w2}c_{w1}u(w2, w1) + c_q^2 u^2(q') \tag{3.6}$$

The first three terms on the right hand side of the equation can be expanded to include the small accuracy and resolution effects from the instruments used.

$$u^2(w2) = u^2(w2_a) + u^2(w2_r) \tag{3.7}$$

$$u^2(w1) = u^2(w1_a) + u^2(w1_r) \tag{3.8}$$

$$u^2(t) = u^2(t_a) + u^2(t_r) \tag{3.9}$$

The uncertainty terms subscripted with *a* represent the accuracy (or systematic) effects of the instruments and terms subscripted with *r* represent the uncertainty from the resolution of the instrument. The fourth term on the right hand side of the equation, the uncertainty of the value used for specific weight, includes the effects of using a value of specific weight that is too large and ignoring the effects of water temperature variation on water density. It can be expanded as:

$$u^2(\gamma) = u^2(\bar{\gamma}) + u^2(\gamma') \tag{3.10}$$

where $\bar{\gamma}$ is the difference between the average "true" specific weight and the value used, and γ' is the uncertainty due to the water temperature variation from the average temperature of the laboratory water.

Equation 3.6 can be rewritten using the expanded equations for the four terms as:

$$u_c^2(y) = c_{w2}^2 u^2(w2_a) + c_{w1}^2 u^2(w1_a) + c_t^2 u^2(t_a) + c_{w2}^2 u^2(w2_r) + c_{w1}^2 u^2(w1_r) + c_t^2 u^2(t_r) + c_\gamma^2 u^2(\bar{\gamma}) + c_\gamma^2 u^2(\gamma') + c_q^2 u^2(q') + 2c_{w2}c_{w1}u(w1, w2) \tag{3.11}$$

The last term in equation 3.11 is the correlated uncertainty resulting from using the same scale for measuring the initial and final weight. The first six right hand terms are type B uncertainties estimated from the manufacturer instrument specification for accuracy and resolution. The uncertainty contributed by the repeatability of the measurement, $u(q')$, is the only standard uncertainty estimated using measured data (Type A). This term includes effects from technician skill, flow steadiness, and the process of diverting the flow during the measurement that are not represented in the other terms and are important contributions to the measurement uncertainty.

Equation 3.11 can be simplified. Because the same scale is used for all the weighing measurements, the weighing terms can be rearranged and combined. The standard uncertainties for weighing are equal, $u(w2_a) = u(w1_a)$ and $u(w_r) = u(w2_r) = u(w1_r)$. The correlated uncertainty (covariance) can be rewritten as $u_{w2w1} = u(w2_a) \times u(w1_a) = u^2(w2_a)$. Using equations 3.2 and 3.3, the sensitivity coefficients for weighing can be written as, $c_w = c_{w2} = -c_{w1}$ and $c_{w2}c_{w1} = -c_w^2$. The correlation of the instrument accuracy (or systematic) uncertainty between the two measurements is negative, and results in the accuracy uncertainties from weighing not contributing to the combined uncertainty of the discharge measurement. The random uncertainty of the two measurements are additive. Equation 3.11 can been rearranged as,

$$u_c^2(y) = 2c_w^2 u^2(w_r) + c_t^2 u^2(t_a) + c_t^2 u^2(t_r) + c_\gamma^2 u^2(\bar{\gamma}) + c_\gamma^2 u^2(\gamma') + c_q^2 u^2(q') \tag{3.12}$$

In equation 3.12, each standard uncertainty, u_i, is computed by:

$$u_i = \frac{a_i}{Divisor} \tag{3.13}$$

where a_i is assumed to be either the limits within which the true value will lie or a standard deviation. The *Divisor* is the value by which a_i can be divided to yield the standard deviation for the probability distribution assumed for the *i*-th source of uncertainty.

A rectangular distribution is assumed for the resolution of weighing and timing and for the accuracy specification of the scales and stop watch. The manufacturer specifications in the absence of information on the probability distribution are assumed to be the limits of their respective probability distributions. Similarly, the range of specific weight due to the effect of variable water temperature is also assumed to have a rectangular probability distribution. The uncertainty due to the systematic error in the specific weight is assumed to have a triangular distribution, because the estimated value of the true specific weight is most likely to be very near the true value. A normal distribution is used for the discharge repeatability term, because this value is based on measured data having a normal probability distribution.

3.4.1 Uncertainty for a discharge measurement determined by a single sample

The estimated uncertainties for discharge measurements of $3.681 x 10^{-4}$ m^3/s, $54.085 \text{ x } 10^{-4}$ m^3/s, and $147.248 \text{ x } 10^{-4}$ m^3/s (0.013 ft^3/s, 0.191 ft^3/s, and 0.520 ft^3/s) obtained from a single weighing and timing of the flow are computed using equation 3.12. Instrument specifications, information on specific weight variation, and existing discharge measurement data are used to estimate the individual standard uncertainties in equation 3.12.

The existing discharge data is typical of what arises in practice. Many older measurements are available but from a number of slightly different conditions than the current measurement. For any given measurement condition only a small number of repeated measurements are made at exactly the same conditions. The data collected at approximately the same discharges, listed in table 3.3, are used to compute pooled statistics for the standard deviation due to measurement repeatability.

Table 3.3 Measured data used for computing pooled statistics for estimating repeatability uncertainties. *[n, number of samples; std. dev., standard deviation]*

Discharge Measurement in m^3/s								
$3.681 x 10^{-4}$			$54.085 \text{ x } 10^{-4}$			$147.248 \text{ x } 10^{-4}$		
Pooled Data in m^3/s			Pooled Data in m^3/s			Pooled Data in m^3/s		
n	Mean	std. dev.	n	mean	std. dev.	n	mean	std. dev.
3	$3.66 x 10^{-4}$	$9.447 x 10^{-6}$	4	$4.94 x 10^{-3}$	$3.396 x 10^{-5}$	5	$1.35 x 10^{-2}$	$1.372 x 10^{-4}$
3	$3.76 x 10^{-4}$	$6.579 x 10^{-6}$	3	$5.59 x 10^{-3}$	$9.578 x 10^{-6}$	5	$1.50 x 10^{-2}$	$1.250 x 10^{-4}$
3	$3.52 x 10^{-4}$	$1.825 x 10^{-6}$	3	$5.40 x 10^{-3}$	$1.199 x 10^{-5}$	5	$1.47 x 10^{-2}$	$2.785 x 10^{-4}$
3	$3.43 x 10^{-4}$	$2.286 x 10^{-6}$	3	$4.94 x 10^{-3}$	$4.486 x 10^{-5}$	5	$1.36 x 10^{-2}$	$5.925 x 10^{-5}$
5	$3.27 x 10^{-4}$	$7.406 x 10^{-7}$	3	$5.33 x 10^{-3}$	$7.048 x 10^{-5}$	5	$1.45 x 10^{-2}$	$8.189 x 10^{-5}$
5	$3.20 x 10^{-4}$	$4.350 x 10^{-6}$	3	$5.06 x 10^{-3}$	$1.650 x 10^{-5}$	5	$1.37 x 10^{-2}$	$2.347 x 10^{-4}$
3	$3.33 x 10^{-4}$	$4.424 x 10^{-6}$	5	$5.11 x 10^{-3}$	$2.429 x 10^{-5}$	7	$1.33 x 10^{-2}$	$1.535 x 10^{-4}$
3	$3.44 x 10^{-4}$	$3.602 x 10^{-6}$	5	$5.78 x 10^{-3}$	$2.161 x 10^{-5}$	7	$1.56 x 10^{-2}$	$2.214 x 10^{-4}$
			3	$5.15 x 10^{-3}$	$1.174 x 10^{-5}$	8	$1.57 x 10^{-2}$	$1.957 x 10^{-4}$
						5	$1.55 x 10^{-2}$	$7.572 x 10^{-5}$
						3	$1.34 x 10^{-2}$	$3.594 x 10^{-5}$
						3	$1.54 x 10^{-2}$	$8.443 x 10^{-5}$
						3	$1.53 x 10^{-2}$	$3.063 x 10^{-5}$

For each discharge, the estimate of the standard deviation of the repeatability at a discharge can be computed from a pooled variance:

$$u^2(q_p^{'}) = \frac{\sum_{k=1}^{M}(n_k - 1)u^2(q_k^{'})}{\sum_{k=1}^{M}(n_k - 1)} \tag{3.14}$$

where M is the number of different conditions, $u^2(q_p^{'})$, is the pooled variance and $u^2(q_k^{'})$ is a variance for the *kth* condition. The pooled standard deviation computed from the square root of the variance, is listed for each discharge for which the uncertainty is to be computed in table 3.4 along with the pooled degrees of freedom and the student's t value for a 95% confidence interval.

Table 3.4 Pooled statistics for the uncertainty due to repeatability.

Discharge m^3/s	Degrees of freedom	Pooled standard deviation m^3/s	Student's t
3.681×10^{-4}	20	3.897×10^{-6}	2.086
54.085×10^{-4}	23	1.688×10^{-5}	2.069
147.248×10^{-4}	53	1.485×10^{-4}	2.000

Tables 3.5 through 3.7 summarize the uncertainty computations for a single measurement (one sample) in m^3/s for three different discharges: 3.681×10^{-4}, 54.085×10^{-4} m^3/s, and 147.248×10^{-4} m^3/s (0.013 ft^3/s, 0.191 ft^3/s, and 0.520 ft^3/s). Table rows 1 thru 6 correspond to each term on the right hand side of equation 3.12. The rightmost column, except for the last two rows, contains the contribution of each term to the combined uncertainty. The combined uncertainty, in table row 7, is computed using equation 2.10 (sec 2).

The uncorrected specific weight value contributes uncertainty from the effects of temperature variation and from the bias in the values used. The uncorrected specific weight mean bias is the difference between the unused value and the most likely true value for specific weight, 9.7712 kN/m^3.

The coverage factor, k, used to compute the expanded uncertainty is dependent on the effective sample size. A coverage factor of 2 is appropriate when the uncertainties are based on large sample sizes $(n \geq 30)$ and a 95% confidence interval. For small sample sizes, student's t distribution is used to determine the coverage factor. The type A standard uncertainty for measurement repeatability has a known sample size. For the type B standard uncertainties used, the sample size is assumed to be infinite.

For the largest discharge, all standard uncertainties used to compute the combined uncertainty are based on large sample sizes. For the two smaller discharges, the effective sample size for the uncertainty contribution from the repeatability term is smaller than 30. The type B standard uncertainties contribute values of 0 to the summation in the denominator. For these situations the effective degrees of freedom are computed using equation 2.13 (sec 2), the Welch-Satterthwaite formula. The only non zero term in the denominator is computed from the pooled statistics from repeated discharge measurements, which are listed in table 3.4. For a discharge of 3.681×10^{-4} m^3/s the effective degrees of freedom from the Welch-Satterthwaite formula is :

$$v_{eff} = \frac{\left(3.89 \times 10^{-6}\right)^4}{\left(\frac{3.89 \times 10^{-6}}{20}\right)} = 20$$

and for 54.085×10^{-4} m^3/s the degrees of freedom are

$$v_{eff} = \frac{\left(1.69\times10^{-5}\right)^4}{\left(\frac{1.69\times10^{-5-6}}{23}\right)} = 23\,.$$

For the effective degrees of freedom, 20 and 23, the student's t values for a 95% confidence interval give the coverage factors, 2.09 and 2.07, respectively.
The expanded uncertainties are computed by multiplying the combined uncertainty with a coverage factor to yield a 95% confidence interval. The expanded uncertainties, in table row 8, are reported to 2 significant figures in units of m^3/s and percent. The quality of the data used to compute expanded uncertainty rarely justifies reporting more than 2 significant figures (UKAS, 2007, pg21).

Table 3.5 Example of computing the uncertainty for: Q=3.681x10^{-4} m^3/s, t= 63.3 s, W_2-W_1=(24.0-0.0)kg.

Source of Uncertainty	a_i,Value	Probability distribution	Divisor	c_i, Sensitivity coefficient	Contribution to uncertainty (m^3/s)
Correlated resolution of 2 weights	0.05 kg	Rectangular	$\sqrt{3}$	1.6109 x10^{-6}	6.5764 x10^{-8}
Stop watch accuracy	0.0058%	Rectangular	$\sqrt{3}$	6.1179 x10^{-7}	1.2968 x10^{-9}
Stop watch resolution	0.01 sec	Rectangular	$\sqrt{3}$	6.1179 x10^{-7}	3.5322 x10^{-9}
Uncorrected specific weight mean bias	35.741 N/m^3	Triangular	$\sqrt{6}$	3.949 x10^{-9}	5.7622 x10^{-8}
Uncorrected specific weight variation due to temperature	19.8802 N/m^3	Rectangular	$\sqrt{3}$	3.949 x10^{-9}	4.5327 x10^{-8}
Repeatability	3.897 x10^{-6} m^3/s	Normal	1	1.0	3.897 x10^{-6}
Combined uncertainty m^3/s					3.89 x10^{-6}
Expanded Uncertainty m^3/s, k=2.09					8.1 x10^{-6} (2.2%)

Table 3.6 Example of computing uncertainty for: Q=54.085 x10^{-4} m^3/s, t=34.6s, W_2-W_1=(187.7-1.0)kg

Source of Uncertainty	a_i,Value	Probability distribution	Divisor	c_i, Sensitivity coefficient	Contribution to uncertainty (m^3/s)
Correlated resolution of 2 weights	0.05 kg	Rectangular	$\sqrt{3}$	2.9471x10^{-6}	1.2032 x10^{-7}
Stop watch accuracy	0.0058%	Rectangular	$\sqrt{3}$	1.5903x10^{-5}	1.8425 x10^{-8}
Stop watch resolution	0.01 sec	Rectangular	$\sqrt{3}$	1.5903x10^{-5}	9.1813 x10^{-8}
Uncorrected specific weight mean bias	35.741 N/m^3	Triangular	$\sqrt{6}$	5.6107x10^{-8}	8.1868 x10^{-7}
Uncorrected specific weight variation due to temperature	19.8802 N/m^3	Rectangular	$\sqrt{3}$	5.6107x10^{-8}	6.4399 x10^{-7}
Repeatability	1.688x10^{-5} m^3/s	Normal	1	1.0	1.688x10^{-5}
Combined uncertainty m^3/s					1.69x10^{-5}
Expanded Uncertainty m^3/s, k =2.07					3.5 x10^{-5} (0.65%)

Table 3.7 Example of computing uncertainty for: Q=147.2476 x10^{-4} m^3/s, t=12.49s, W_2-W_1=(185.1-1.2)kg

Source of Uncertainty	a_i,Value	Probability distribution	Divisor	c_i, Sensitivity coefficient	Contribution to uncertainty (m^3/s)
Correlated resolution of 2 weights	0.05 kg	Rectangular	$\sqrt{3}$	8.1642x10^{-6}	3.3330 x10^{-7}
Stop watch accuracy	0.0058%	Rectangular	$\sqrt{3}$	1.202x10^{-4}	5.0273 x10^{-8}
Stop watch resolution	0.01 sec	Rectangular	$\sqrt{3}$	1.202x10^{-4}	6.9397 x10^{-7}
Uncorrected specific weight mean bias	35.741 N/m^3	Triangular	$\sqrt{6}$	1.5309x10^{-7}	2.2338 x10^{-6}
Uncorrected specific weight variation due to temperature	19.8802 N/m^3	Rectangular	$\sqrt{3}$	1.5309x10^{-7}	1.7571 x10^{-6}
Repeatability	1.485x10^{-4} m^3/s	Normal	1	1.0	1.485x10^{-4}
Combined uncertainty m^3/s					1.48x10^{-4}
Expanded Uncertainty m^3/s, k =2.00					2.97 x10^{-4} (2.0%)

The combined uncertainty for a single discharge measurement, computed in tables 3.5 through 3.7, is plotted in figure 3.2 as a function of discharge. The plot shows that the uncertainty in m^3/s of an individual discharge measurement increases with discharge. The combined uncertainty is dominated by the uncertainty contribution from the repeatability of the discharge measurement.

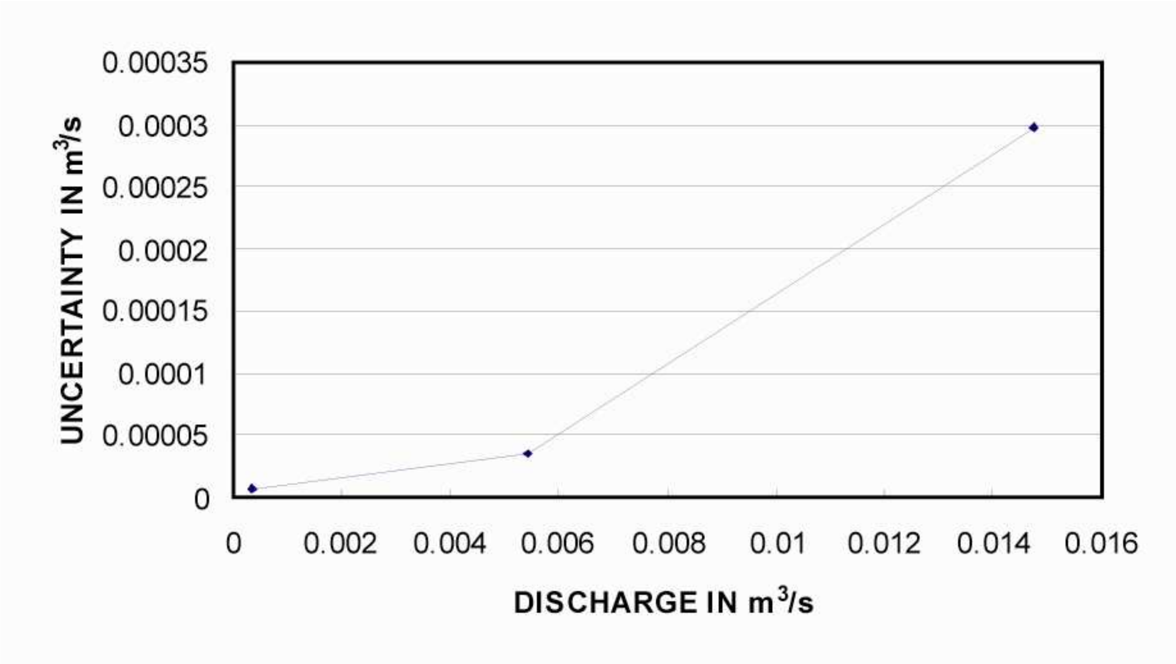

Figure 3.2 Uncertainty for laboratory discharge by weighing.

3.4.2 Uncertainty for discharge measurement determined by three samples

Multiple samples or measurements at the same discharge can be used to reduce the uncertainty of the discharge measurement when there is significant variation in repeated measurements. When multiple samples are used, the measured discharge is the average of the repeated discharge measurements. The standard uncertainty contributed by the repeatability to the discharge measurement is reduced by a factor of $1/\sqrt{n}$ where n is the number of individual measurements made at a set discharge (UKAS, 2007, pg 19). Table 3.8 lists the computed uncertainties of the discharge measurement when 3 measurements (or samples) are made at a set discharge. Uncertainty of the measured discharge is reduced by approximately 30% when 3 repeated measurements are averaged (figure 3.3).

Table 3.8 Effect of repeating the discharge measurement 3 times on discharge uncertainty.

Discharge (m^3/s)	Number of measurements	Standard uncertainty for repeatability (m^3/s)	Combined uncertainty (m^3/s)	Expanded uncertainty (percent)
3.681×10^{-4}	3	2.250×10^{-6}	2.250×10^{-6}	1.3
54.085×10^{-4}	3	9.75×10^{-6}	9.8×10^{-6}	0.49
147.2476×10^{-4}	3	8.57×10^{-5}	8.58×10^{-5}	1.2

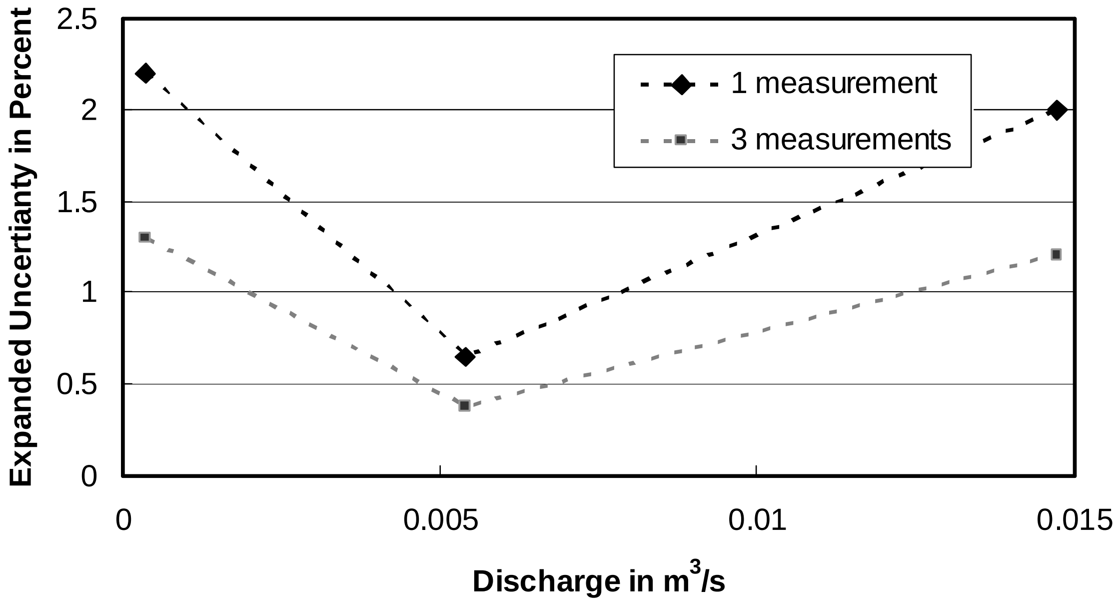

Figure 3.3 Effect of increasing number of measurements made at a discharge on expanded uncertainty.

3.5 Summary

The uncertainty analysis can be used to estimate the uncertainty of a laboratory discharge measurement using the weighing and timing method. The GUM method uses Type A and Type B classifications of uncertainty instead of random and systematic. However, the different terminology does not alter the methodology or result that organizations such as AIAA and ASME have adopted. To determine the individual standard uncertainties, information on the measurement process and instrumentation used is needed. Repeated measurement data is also helpful in determining the uncertainty due to random effects but can be estimated by other means if necessary. The discharge measurement example demonstrates that uncertainty can be computed using a combination of measurement data and information from instrumentation specifications and laboratory conditions. It also illustrates that repeated measurements can be used to reduce the uncertainty of the final (or averaged) measurement.

3.6 References

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