Standardized Uncertainty Analysis Frameworks for Hydrometry: Example #3

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Determination of the discharge in a circular sewer pipe

In this example, the uncertainty in the discharge measured in a circular sewer pipe is calculated using GUM framework (GUM, 1993) and MCM (JCGM 101, 2008). For the sake of brevity only salient calculations are presented.

Measurement situation

Consider a sewer network circular pipe with a radius, *R* (m) equipped with a piezoresistive sensor (Milltronics NivuBar) providing the water level *h*, (m) and a Doppler sensor (Milltronics DEK-B EX/30) measuring the flow velocity, *U* (m/s) through the pipe cross-section *S* (m^2), as shown in Figure 1. It is assumed that the pipe is circular and that there are no deposits on the pipe invert.

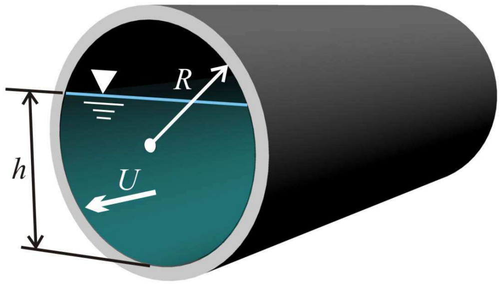

Figure 1. Schematic view of the circular pipe

The discharge *Q* (m^3/s) is then given by the following functional relationship (Bertrand-Krajewski and Muste, 2008)

$$Q(R,h,U) = S(h)U = R^2\left[\text{Arccos}\left(1-\frac{h}{R}\right)-\left(1-\frac{h}{R}\right)\sin\left(\text{Arccos}\left(1-\frac{h}{R}\right)\right)\right]U \qquad (1)$$

The radius, *R* is an input variable for which repeated measurements were acquired in situ. There are no repeated measurements for the water level and the mean flow velocity in the sewer pipe. Described below are the procedures used to estimate the standard uncertainties in *R*, *h* and *U*, respectively $u(R)$, $u(h)$ and $u(U)$ using the available information.

Estimation of standard uncertainty in the pipe radius. The average pipe radius *R* was calculated using four measurements of the diameter *D* carried out randomly at various radial positions in the pipe cross section. The measurements were respectively: 1002, 1000, 997, and 1002 mm. The resulting mean value for *R* is 500.12 ≈ 500 mm. The standard deviation of the four measurements is 1.18 mm. The standard uncertainty $u(R)$ based on $v = n - 1 = 3$ degrees of freedom is then $u(R) = 0.94$ mm ≈ 1 mm. Consequently, the mean value of the radius is $R = 0.5$ m with $u(R) = 0.001$ m.

Estimation of standard uncertainty in h. The in situ water level measurements were measured with a piezoresistive sensor. The sensor was previously calibrated in the laboratory and details on the calibration procedure are presented in Bertrand-Krajewski and Muste (2008). In essence, the calibration used a Perspex column with a class II certified metallic meter as reference, with a standard uncertainty of 0.5 mm as specified by the manufacturer. A total of 60 simultaneous measurements were acquired with the metallic meter and the piezorestitive sensor for water levels varying between 0 and 2 m. Using a least squares regression applied to the measurements collected at five calibration points, the calibration function, with a residual variance $s_l^2 = 0.344398$, was obtained as

$$y = a + bx = 0.508854 + 1.000395\ x \quad (2)$$

where x is the standard true value and y is the value given by the sensor.

Using Equation (2) was then possible to relate a piezoresistive sensor measurement y_0 into the corresponding most likely true value of the water level x_0, and also to evaluate its standard uncertainty $u(x_0)$. For example, a reading of y_0 = 701 mm would most likely correspond to:

$$x_0 = \frac{y_0 - a}{b} \approx 700.2\ mm \quad (3)$$

The standard deviation $s(x_0)$ of the piezoresisitive sensor is the result of two independent contributions: i) the uncertainty in the measured value y_0, and ii) the uncertainty in the calibration curve expressed by the uncertainties in both coefficients $s(a)$ and $s(b)$. $s(x_0)$ is calculated by:

$$s(x_0)^2 = \frac{s_l^2}{b^2}\left(1 + \frac{1}{N} + \frac{(x_0 - \bar{x})^2}{\sum_i n_i (x_i - \bar{x})^2}\right) = 0.354 \quad (4)$$

Accordingly, adopting $u(x_0)$ as $s(x_0)$ results in a value of $u(x_0) \approx 0.6$ mm. The 95 % confidence interval (with a coverage factor of 2) for x_0 is then given by $[x_0\text{-}2u(x_0),\ x_0\text{+}2u(x_0)] \approx$ [699.0, 700.4mm]. The final result of the measurement of y_0 = 701 mm is $x_0 = 700.2 \pm 1.2$ mm. Figure 2 summarizes the calibration results with the solid line representing the first-order regression of the mean values of the calibration points and the associated uncertainty intervals estimated at 95 % confidence level.

The above results are valid for measurements taken under well controlled laboratory conditions. For measurements taken in situ in an actual sewer, the measurement standard uncertainty is larger than 0.6 mm because the water level is not flat and stable due to small free surface waviness produced by internal (turbulence) and external perturbations (such as the air currents in the channel). An estimate of this additional source of uncertainty based on observations made in various measurement locations is evaluated to be about u_r = 5 mm.

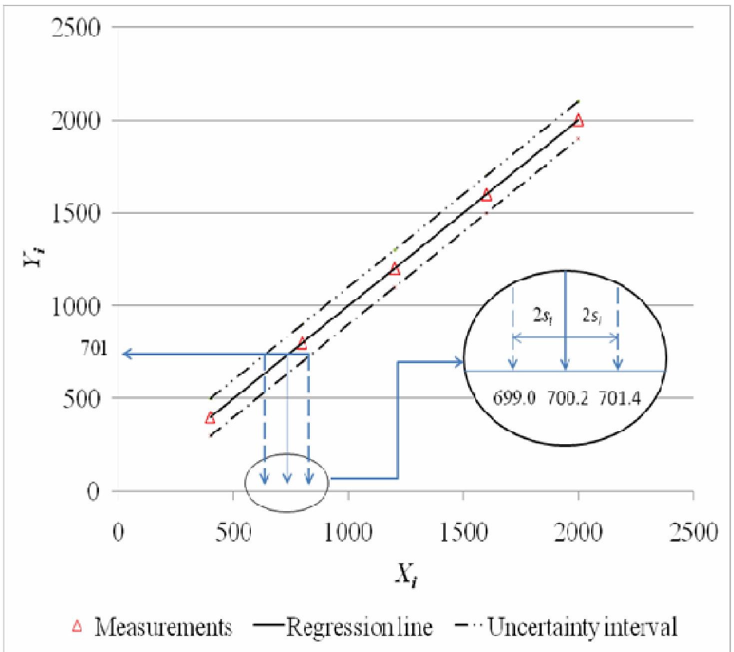

Figure 2. Results of the calibration results for the piezoresistive sensor for the 0 to 2 m measurement range

The in situ measurement standard uncertainty $u(h)$ for the piezoresistive sensor is obtained by summing the root mean square of the contributing uncertainties

$$u(h) = \sqrt{u(x0)^2 + u_r^{\ 2}} = \sqrt{0.6^2 + 5^2} = \sqrt{0.36 + 25} \approx 5mm$$

These variables need to account for additional uncertainty components if the measurements are taken during storm events whereby flow unsteadiness and turbulence levels are considerable. Preliminary estimates indicate a value for $u(h)$ as high as 15 mm can be obtained for such situations.

Estimation of standard uncertainty in U. The mean flow velocity U is measured with the Doppler sensor. As no proper calibration of the sensor was feasible, $u(U)$ has been estimated by direct comparison with discharge measurements carried out with a mobile electromagnetic velocimeter of traceable accuracy. Based on this comparison (considered as Type B with infinite degrees of freedom) was deemed that $u(U)$ = 0.05 m/s. The above estimation, however, can be further refined if needed for a more rigorous analysis. The limitation of the previous assessment stems from the fact that the velocity is not measured over the pipe cross section, but locally in a point. If resources are available (which was not the case in the present situation), an index velocity obtained in the controlled conditions would be required to improve the uncertainty estimation. The additional tests would need to cover a range of depths and flow velocities through the pipe. The result of the additional test would provide a correction coefficient and associated uncertainty. The uncertainty estimation accounting for the effect of velocity distribution in the corss section can potentially exceed the magnitude of the error associated with the measurement of the raw velocity.

With the above estimates available the total uncertainty in the results can be determined. The following values are used im the uncertainty propagation equation: R = 0.5 m, $u(R)$ = 0.001 m; h = 0.7 m, $u(h)$ = 0.005 m; U = 0.8 m/s, $u(U)$ = 0.05 m/s. Using equation (1) in conjunction with the mean values of the input variables one obtains Q = 0.4697 m^3/s. According to GUM framework (see Section 3.1), the combined standard uncertainty and the expanded uncertainty are given by:

$$u_c(Q) = \sqrt{u(R)^2\left(\frac{\partial Q}{\partial R}\right)^2 + u(h)^2\left(\frac{\partial Q}{\partial h}\right)^2 + u(U)^2\left(\frac{\partial Q}{\partial U}\right)^2} \tag{5}$$

$$U(Q) = ku_c(Q) \tag{6}$$

The input variables are assumed uncorrelated. The effective degrees of freedom for the measureand are obtained using the corresponding degrees of freedom for each variable ($v_R = 3, v_h = 59, \text{and } v_h = \infty$). The resulting effective degree of freedom for the measurand is 70256, a large value that can be considered for practical purposes as an infinite value. Finally, using equation (6), the expanded uncertainty based on *t*-distribution (at 95 % level of confidence) provide k = 2, which subsequently gives $U(Q)$ = 0.0592 m^3/s, equivalent to $\pm\,12.6\%$ of the total value.

The numerical values in Equation (5) provide a small values for the first term (= 7.26.10-7 m^3/s) compared to the two other ones, indicating as expected that the contribution of the uncertainty of the pipe radius R to the total uncertainty in Q is negligible and can be ignored. Using the same type of inference can be noted that the contribution of the uncertainty in depth is lower than that of the uncertainty in velocity for this given set of values. This conclusion can however be drawn for all possible values of R, h and U. Separate analyses need to be made for each particular set of values (R, h, U) to evaluate the relative contribution of the uncertainties of individual variables to the total measurand uncertainty.

The results calculated above indicate that for the discharge value of $\overline{Q}$ = 0.4697 m^3/s, the 95 % confidence interval uncertainty is (0.4106, 0.5289). The calculated uncertainties are dependent on the mean values where they are estimated as the sensitivity coefficients are different over the measurement range of the input variables. Similar estimation carried over the time series of actual measurements in the sewer will indicate that the total uncertainty of the discharge measurements varies as shown in Figure 3.

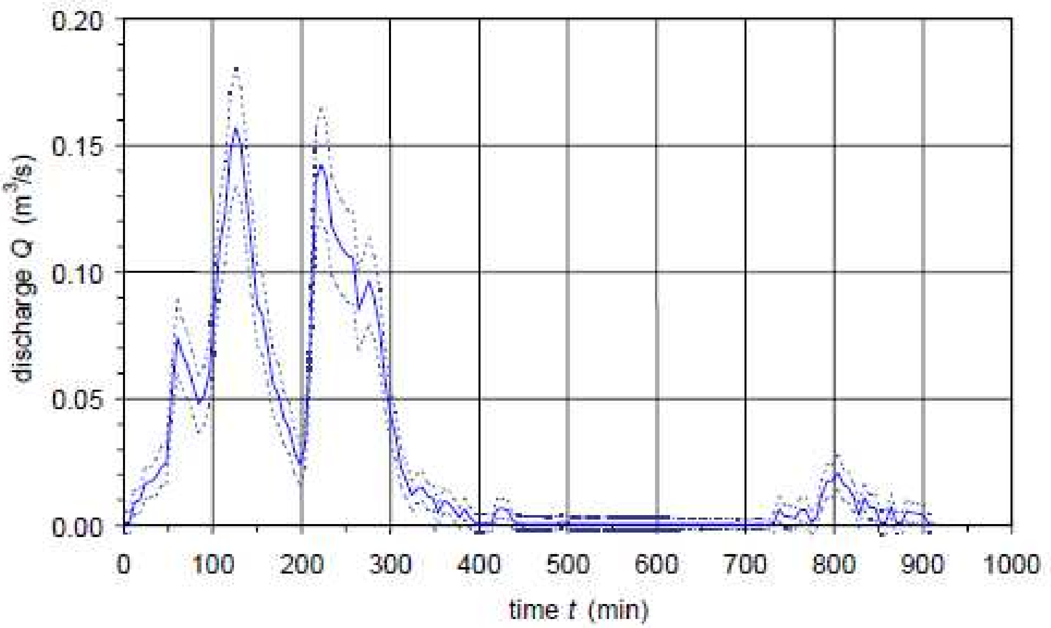

Figure 3. Measured hydrograph in a sewer system during a storm event and its 95 % confidence interval uncertainty estimates

REFERENCES

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