First meeting of the Management Committee (MC) for the CHy Project on the Assessment of the Performance of Flow Measurements and Techniques for the Period 2013-2016
Geneva, 2 to 5 December 2013

Uncertainty Analysis of Stage-Discharge Relations using the BaRatin Bayesian Framework

Jérôme LE COZ¹, Benjamin RENARD¹, Laurent BONNIFAIT², Flora BRANGER¹, Raphaël LE BOURSICAUD¹

¹ Irstea Lyon, Hydrology-Hydraulics research unit, Lyon, France
² DRIEA / CETE Ile-de-France, Trappes, France

Rev. Thomas Bayes (~1702 - 1761)
Robert Manning (1816 - 1897)
Rating curves are used at most hydrometric stations

A rating curve is a one-to-one stage-discharge relation $Q(h)$ which prevails at a given flow section in some reference conditions.

It is used to convert stage recordings into discharge time series.

→ How to establish it practically? What are the related uncertainties?
Streamgaugings

A rating curve is usually built using gaugings, i.e. direct measurements of stage and discharge at some moments.

The uncertainty of gaugings is variable according to the measurement technique and environmental conditions.
The issue of rating shifts

The stage-discharge relation may suddenly or gradually change due to bed evolution, vegetation growth, ice processes, variable backwater, transient flow hysteresis, etc.

As a first step, such rating shifts are ignored, and only stationary rating curves are addressed.
Requirements for a rating curve methodology

A hydraulic basis
→ *interpretation of controls*
→ *extrapolation to ungauged parts*

A statistical approach
→ *fit to uncertain gaugings*
→ *sound uncertainty analysis*

A user-friendly tool
→ *as simple as possible*
→ *soft and documentation*
Hydraulic controls

The physical characteristics of a channel which govern the relation between the water stage and the discharge at a given location within the channel

2 different types of hydraulic controls in rivers and canals:

Channel control

Section control
Hydraulic controls successively activate and disappear.

Example of section and channel hydraulic controls changing with discharge for a typical hydrometric station with a gravel-bedded channel and a floodplain.
Hydraulic controls can be approximated using simple power functions with 3 parameters:

\[ Q = a(h - b)^c \]

**Manning-Strickler**

\[ Q = KB \sqrt{J} \left( h - h_0 \right)^{5/3} \]

(wide rectangular channel, Uniform, steady flow)

**Rectangle weir / natural riffles**

\[ Q = CB \sqrt{2g} \left( h - h_0 \right)^{3/2} \]

**Triangle weir / V-notch**

\[ Q = C \tan(\alpha) \sqrt{2g} \left( h - h_0 \right)^{5/2} \]

**Orifice**

\[ Q = C_o A_o \sqrt{2g} \left( h - h_0 \right)^{1/2} \]
Defining the structure of hydraulic controls with BaRatin

\[
Q = \sum_{r=1}^{N_{\text{segments}}} l_{\kappa_{r-1}, \kappa_r}(h) \sum_{j=1}^{N_{\text{controls}}} M(r, j) a_j (h - b_j)^{c_j}
\]

- Transition levels between segments
- Matrix of the hydraulic controls
- Parameters \( a, b, c \) of the hydraulic controls

<table>
<thead>
<tr>
<th>From (m)</th>
<th>To (m)</th>
<th>+/- (m)</th>
<th>Control1</th>
<th>Control2</th>
<th>Control3</th>
<th>Control4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
M = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
\end{pmatrix}
\]
The individual uncertainty of every gaugings must be assessed.

The conceptual uncertainty (remnant error) accounts for the non-parametric error, i.e. due to the assumed structure of the stage-discharge function.

Distributions other than Gaussian can also be assumed, if necessary.
Bayesian inference

Bayes theorem is applied to infer the posterior distribution:

\[
p(\theta, \sigma_f | \tilde{H}, \tilde{Q}) \propto p(\tilde{Q} | \theta, \sigma_f, \tilde{H}) p(\theta, \sigma_f)
\]

Input data:

- Hydraulic control segments and laws (priors)
- Prior distributions of parameters, \( \theta \)
- Gaugings and associated uncertainties

\[
H_i, \tilde{Q}_i, u_i^O
\]
Monte Carlo Markov Chains (MCMC) simulations ($\times 10^4$) of the posterior distribution.

Posterior distributions of parameters $\theta$ and of $\sigma_f$. 

MCMC sampling of the posterior distribution.
Compound section controls: Arc river at Pontamafreya

Combination of 3 artificial broad-crested horizontal weirs

No backwater effects even for very high flows

Relatively high approach velocities
Compound section controls: Arc river at Pontamafrey

More uncertain flood gaugings using image-based analysis (LSPIV)

Posterior values are consistent with prior knowledge and adjusted to the gaugings.

However, discharge coefficients of the weirs are significantly higher than expected, which may be related to the high velocities of the approaching flow.

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$c_1$</th>
<th>$\kappa_2$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$c_2$</th>
<th>$\kappa_3$</th>
<th>$a_3$</th>
<th>$b_3$</th>
<th>$c_3$</th>
<th>$\sigma_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior</td>
<td>21.3</td>
<td>-0.12</td>
<td>1.50</td>
<td>0.38</td>
<td>37.4</td>
<td>-</td>
<td>1.50</td>
<td>0.88</td>
<td>25.7</td>
<td>-</td>
<td>1.50</td>
<td>-</td>
</tr>
<tr>
<td>Std.</td>
<td>2</td>
<td>0.1</td>
<td>0.025</td>
<td>0.1</td>
<td>3.5</td>
<td>-</td>
<td>0.025</td>
<td>0.15</td>
<td>2.5</td>
<td>-</td>
<td>0.025</td>
<td>-</td>
</tr>
<tr>
<td>Max</td>
<td>23.1</td>
<td>-0.16</td>
<td>1.50</td>
<td>0.35</td>
<td>45.5</td>
<td>0.35</td>
<td>1.51</td>
<td>0.82</td>
<td>28.6</td>
<td>0.82</td>
<td>1.52</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Compound channel controls: Ill river at Ladhof

Cross-stream distance [m]

Gaugings

Elevation in staff gauge datum [m]

<table>
<thead>
<tr>
<th>Segment</th>
<th>From (m)</th>
<th>To (m)</th>
<th>+/- (m)</th>
<th>Control1</th>
<th>Control2</th>
<th>Control3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>weir</td>
<td>MC</td>
<td>FP</td>
</tr>
<tr>
<td>Segment2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Segment3</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Control1 | a1 | 50 | +/- | 20 |
| Control2 | a2 | 28.5 | +/- | 9 |
| Control3 | a3 | 9.5 | +/- | 5 |
Compound channel controls: Ill river at Ladhof

Streamwise profile of the 1-D hydraulic model (MAGE) used to compute hydraulic priors for the main channel (MC) and floodplain (FP) controls.

Debord equation was applied but acceptable results could be reproduced using divided rectangle channels.
Compound channel controls: Ill river at Ladhof

Posterior values are consistent with prior knowledge and adjusted to the gaugings.

From very wide priors, the information contained in gaugings led to a quite narrow posterior distribution, except for low flows due to the scatter of the data.

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$c_1$</th>
<th>$\kappa_2$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$c_2$</th>
<th>$\kappa_3$</th>
<th>$a_3$</th>
<th>$b_3$</th>
<th>$c_3$</th>
<th>$\sigma_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior value</td>
<td>50</td>
<td>0.8</td>
<td>1.50</td>
<td>2</td>
<td>28.5</td>
<td>-</td>
<td>1.67</td>
<td>3</td>
<td>9.5</td>
<td>-</td>
<td>1.67</td>
<td>-</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>10</td>
<td>0.25</td>
<td>0.025</td>
<td>0.5</td>
<td>4.5</td>
<td>-</td>
<td>0.025</td>
<td>0.5</td>
<td>2.5</td>
<td>-</td>
<td>0.025</td>
<td>-</td>
</tr>
<tr>
<td>Max Post</td>
<td>39.0</td>
<td>0.86</td>
<td>1.50</td>
<td>1.66</td>
<td>21.0</td>
<td>0.47</td>
<td>1.65</td>
<td>3.19</td>
<td>10.9</td>
<td>3.19</td>
<td>1.66</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Conclusions

Stage-discharge relations used at hydrometric stations are based on combinations of hydraulic controls, which can be represented by a mixture of power functions established for reference hydraulic conditions, i.e. a stationary rating curve.

- formalizing the hydraulic knowledge of hydrometric station managers, including hydraulic modelling results

The Bayesian analysis of prior hydraulic controls and gaugings with individual uncertainties is a convenient tool for deriving rating curves and their uncertainty at a given level of confidence (usually 95%).

- quantifying uncertainty on rating curves

To assess the uncertainty of instantenous discharges, one must consider not only the stationary rating curve uncertainty, but also the uncertainty due to temporary or permanent rating shifts, as well as the uncertainty on the water stage recordings.

- providing uncertainty on hydrological data
Conclusions

A useful tool for assessing the uncertainty improvement thanks to non-intrusive, more uncertain flood discharge data.
Perspectives for BaRatin

Sharing the method and software

- Free release under Irstea license (open-source release intended)
- Windows / Linux binaries on public forge
- Documentation in French (done) and in English (soon)
- Interface languages: French, English, Spanish, Portuguese, Romanian, etc.
- Integration into BAREME software (French national hydrometric services)

→ write to baratin.dev@lists.irstea.fr to get the free software

Further development of the method and software

- Different options for the conceptual uncertainty → $\sigma_f$ as a function of $Q$
- Propagation of uncertainties in the discharge time series, averages and flood/drought percentiles
- **Managing rating shifts** (PhD on non stationary rating curves):
  - Discontinuous or continuous shifts due to bed changes
  - Hysteresis due to transient flow effects
  - Variable backwater
  - Seasonal vegetation growth, ice processes...
Perspectives for BaRatin

**Managing rating shifts** (PhD on non stationary rating curves):
- Discontinuous or continuous shifts due to bed changes
- Hysteresis due to transient flow effects
- Variable backwater
- Seasonal vegetation growth, ice processes...

Rating shifts visible in streamgaugings after each flood, due to morphodynamical changes of the low-flow natural control (Ardèche river at Barutel)
Perspectives for BaRatin

Managing rating shifts (PhD on non stationary rating curves):
- Discontinuous or continuous shifts due to bed changes
- Hysteresis due to transient flow effects
- Variable backwater
- Seasonal vegetation growth, ice processes...

Episodic streamgauging deviations to the reference rating curve, due to transient flow conditions (hysteresis) in the Ebro river at Asco station
Perspectives for BaRatin

Managing rating shifts (PhD on non stationary rating curves):
- Discontinuous or continuous shifts due to bed changes
- Hysteresis due to transient flow effects
- Variable backwater
- Seasonal vegetation growth, ice processes...

Annual cycles of streamgauging deviations to the reference rating curve, due to the growth of seasonal aquatic vegetation