CHy Statement on the Terms Stationarity and Nonstationarity

Introduction

As climate change became an increasingly prominent topic in both scientific and public discourse over the past two decades, technical terms like "stationarity" and "nonstationarity" also became more conspicuous. Historically, such terms arose in the mathematics literature, and have been commonly used in engineering and related disciplines. Indeed, stationarity and nonstationarity have traditionally had important implications for water resources engineering and management. As such, they tended to be used in precise and rigorous terms.

Recently, however, the term nonstationarity has been used to imply a process behavior that is synonymous, uniquely, with change and, by inference, with climatic change. The much cited commentary by Milly et al. (2008) entitled "Stationarity is Dead: Whither Water Management?" is one such example. Such usage has resulted in considerable misunderstanding because, in reality, stationarity and nonstationarity cannot be distinguished based merely on data, except where changes in an underlying process are so dramatic that no statistical assessment is necessary (Koutsoyiannis, 2011a). Given that the concepts of stationarity and nonstationarity are critical to the practical field of water resources planning and design, it is essential that the hydrological community have a sound and thorough understanding of what these terms refer to. This note attempts to clarify the fundamentals.

Before proceeding, it is necessary to acknowledge a basic truth: all natural systems undergo change, unequivocally and unconditionally. Events such as the Big Bang, supernovae, and planetary motions guarantee this truth. The issue, therefore, is not whether hydroclimatic systems exhibit stationary or nonstationary behavior but, rather, with how we answer the following question: is the change in the system substantial enough to *require* a complex deterministic characterization of the process, or can a comparatively simple stationary stochastic model accurately represent the process? Answering this question is not a purely academic exercise; it has significant implications for the practice of water resources planning and design.

Stationarity and Nonstationarity Defined

Stationarity is a property of an underlying stochastic process, and not of observed data. Kendall *et al.* (*The Advanced Theory of Statistics*, 1983) describe it as:

Let u_t, t= . . . , -1, 0, 1, . . . , be the random variables describing successive terms of a time series. Further, let the distribution of any set of *n* consecutive *u*'s, say u_{t+1}, u_{t+2}, . . . , u_{t+n}, be

$$F(u_{t+1}, u_{t+2}, \ldots, u_{t+n}).$$

Then, if F is independent of t for all integral $n>0$, the time series is *strictly stationary*. That is, the joint distribution of any set of n consecutive variables is the same, regardless of where in the series it is chosen.

In practice, we have only a sample size of one from the distribution F; namely, the observational record or “realization” and, in order to use it to draw inferences, it is necessary to make the additional assumption of ergodicity. In other words, we must assume that the time average of one sequence of events from the realization is the same as the ensemble average.

Importantly, however, the Kendall *et al*. description also implies that the joint distribution of any set of n $\underline{u}$'s (not necessarily consecutive) depends only on their *relative* positions in the series. Realizations from stationary processes can exhibit excursions and trends that persist for decades or centuries (Cohn and Lins, 2005). This is a critical and commonly misunderstood characteristic of stationary processes. It means that a finite realization from a stationary stochastic process is not tightly constrained, and that it can appear indistinguishable from one produced by a nonstationary deterministic process.

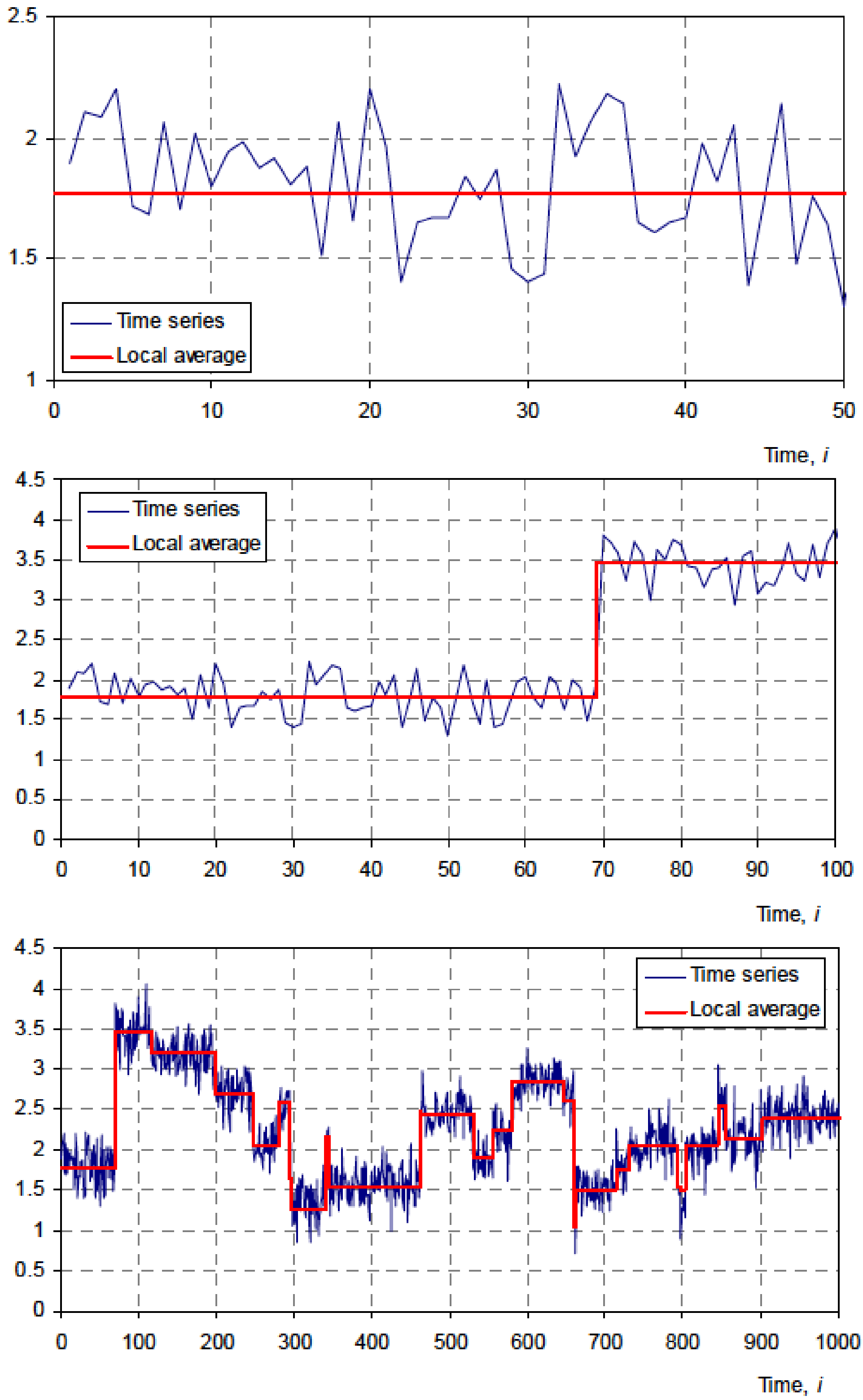
In contrast, nonstationarity can simply be defined as the absence of stationarity; processes that are not stationary have statistical properties that are deterministic functions of time. Demonstrating nonstationarity is more complex than stationarity because it is necessary to do so through analysis of the process physics, chemistry, biology, and ecology.

With respect to hydrological processes over multidecadal to century time scales, climatic nonstationarity appears to be a relatively insignificant source of variablity in comparison with other known sources of nonstationarity such as the building of dams and land use change. As Villarini et al. (2009) point out with respect to flood peaks, it may be easier to claim nonstationarity than to prove it through analyses of actual data.

Discussion

To illustrate the challenge of choosing appropriate modelsl, Koutsoyiannis (2011a) developed a graphical example that uses a synthetic time series with temporal properties typical of hydroclimatic processes. The upper panel of the graphic depicts the first 50 terms of the time series. The sequence exhibits considerable temporal variability although, in the aggregate, one could assume that the series has a constant mean and a constant standard deviation, that is, it is stationary.

The middle panel of the graphic depicts 100 terms of the time series. In this sequence, it is easy to identify two periods separated by a transition (step change) at about the 70th term. The mean for the first period is about 1.8 and for the second period about 3.5. In this instance, some might assume that the time series is nonstationary.



Source: Koutsoyiannis, 2011a.

The lower panel depicts 1000 terms in the time series. It exhibits considerable temporal structure with trends and cyclic swings at various frequencies. By now, it should be evident that the upper and middle panels represent the first 50 and 100 terms in the 1000 term sequence and, more importantly, that the sequence might have been generated by a stationary model. As Koutsoyiannis describes it, "This model consists of the superposition of: (1) a stochastic process, with values m_j derived from the normal distribution $N(2, 0.5)$, each lasting a period τ_j exponentially distributed with $E[\tau_j] = 50$ (the thick line with consecutive plateaus); and (2) white noise, with normal distribution $N(0, 0.2)$. Nothing in this model is nonstationary and, clearly, the process of our example is stationary. In fact, shifting mean models such as the one above have been suggested in the water literature by several researchers (e.g., Potter, 1976; Salas and Boes, 1980; Klemes,1974; Sveinsson et al., 2003)."

Distinguishing stationarity from nonstationarity in this example is a matter of answering a simple question: Does the thick red line of plateaus in the lower panel of the graphic represent a known (deterministic) function or an unknown (random) function? If we are confident that we understand perfectly the causal mechanisms responsible for all of the transitions, then we can adopt a nonstationary description. If, on the other, we admit that we are uncertain as to why the observed variations occurred, then we should use a stationary description.

A fundamental component of engineering design and practice involves predicting or characterizing future conditions with sufficient precision that the consequences of design choices can be evaluated. For example, spillways are designed with the intent of safely passing the largest flood that will occur during the future life of the project. We need to estimate that flood. The traditional approach for characterizing future events is to assume that the characteristics of future events will resemble the past and that the past can be represented by a sample of observations drawn from the same physical process from which the future will be generated (i.e., stationarity). In a statistical sense, while the future will not repeat the past, its properties can be inferred from the past. In some cases this assumption is not valid. For example, urbanization may double the magnitude of 100-year-flood peaks (Moglen and Shivers, 2006), a phenomenon that has been observed in urban basins across much of the United States (Konrad, 2003) and around the world. Consideration of such well-understood nonstationarities is important and clearly appropriate.

However, what about the hypothesized climate-related nonstationarities? This is not so easy, at least with respect to those associated with hydrologic processes such as flood generation. We do not understand the processes, and existing data simply do not reveal a substantial effect. An examination of flood records corresponding to undeveloped watersheds over the past 60 years shows clusters of trends going in both directions, but no consistent trend overall (Lins and Cohn, 2011). Lacking both accurate physical understanding and statistical evidence, it is hard to justify admitting nonstationarity into rigorous analyses.

This is particularly so given the ability of stationary stochastic models to capture the time histories of hydroclimatic processes. Hurst's (1951) observation that Nile streamflows, though seemingly stationary, exhibited persistent excursions from their mean value was the first clear characterization of long-term persistence (LTP) in nature. Nearly a decade earlier, however, Kolmogorov (1940) had formulated the mathematical basis of the "Hurst phenomenon." Subsequently, Mandelbrot found LTP everywhere he looked, most notably in large-scale natural processes, and coined the word "fractals" to describe the entire class of self-similar phenomena. Most recently, in a sequence of papers spanning the past decade, Koutsoyiannis (2002; 2006; 2007; 2010; 2011a,b) recast these earlier contributions into a coherent framework that he termed Hurst-Kolmogorov (HK) dynamics that is defined as

$$\sigma^{(k)} = k^{H-1} \sigma$$

where $\sigma^{(k)}$ is the standard deviation for any (arbitrary) time scale k, $\sigma \equiv \sigma^{(1)}$ is the standard deviation at time scale $k=1$, and H the Hurst coefficient, which in positively dependent processes ranges from 0.5 to 1. Fluctuations at multiple temporal or spatial scales, which may indicate HK stochastic dynamics, are common in nature, such as in turbulent flows, large scale meteorological systems, and even human-related processes.

For $0.5 \leq H < 1$, the HK model is stationary, simple, parsimonious, inexpensive, and transparent in that it does not mask uncertainty.

Conclusion

Although it is important to recognize that change is a characteristic of the natural world, it is also important to acknowledge that the variations recorded in the observational and proxy records of hydroclimatic processes can be represented with stationary stochastic models. In conclusion, there are two critical points to remember from this note:

- Stationary ≠ static
- Nonstationary ≠ change (or trend)

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