Deterministic and Probabilistic prediction approaches in Seasonal to Inter-annual climate forecasting

Christopher Oludhe
Department of Meteorology
University of Nairobi
P. O. Box 30197, Nairobi
KENYA
Email: coludhe@uonbi.ac.ke

1.0 Introduction

Many countries in Africa are prone to extreme weather and climate events and these events have in the past had major negative impacts on various sectors of the economy including agriculture, health, food security, energy and other key social and economic sectors of these countries. Floods and drought has been a common feature in the recent past resulting in massive destruction of property, loss of life, diseases, and food shortages among many other socio-economic miseries.

Recent advances in the science of weather and climate prediction and in particular, seasonal to interannual prediction has made it possible to predict climate with improved accuracy in a time-spans ranging from seasons to over one year in advance. Such knowledge can successfully be used to enhance agricultural production and minimise loss of life and property as well as provide critical information for decision-making.

The objective of this presentation is to highlight some of the approaches to deterministic and probabilistic seasonal to interannual climate prediction applicable to Africa. We shall briefly start by introducing some basic concepts of probability and statistics.
2.0 Basic Ideas on Probability and Statistics

In dealing with climatic data, one usually handles only a small proportion of all the possible values of interest referred to as a *sample*. If all the values of interest were to be examined, then this would constitute what is referred to as the *population* or universe. In majority of the cases, it is not possible to examine the whole population but instead examine a sample. *Statistics* are tools that allow the sample data to be analysed and make inferences (decisions) concerning the entire population.

Some of the important statistical parameters that can be obtained from sample data include: Mean, Variance and Standard Deviation among others. Also of interest are the computations of the various partitions of a given ranked data set, e.g. Terciles, Quartiles and Percentiles among others. In the case of quartiles, if the data are sorted in ascending order, then, the First Quartile (Q1) is a value that has \( \frac{1}{4} \) of the observations at or below its value. We shall see in the later sections that partitioning ranked data into terciles categories are increasingly becoming important in the presentation of seasonal forecasts.

2.1 Understanding probabilities

Probability theory was developed for use in predicting the long-term outcomes of games of chances, e.g., tossing coins, rolling dices, drawing cards e.t.c. In such games, the outcome for a specific trial is uncertain; however, each possible outcome would appear with a certain long-term regularity. The prediction of this regularity is the concern of probability theory. The probability of an event occurring is defined as the ratio of the number of possible outcomes in an event to the total number of outcomes in the sample space.

\[
\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of cases}}
\]

These basic probability ideas have found application in weather and climate forecasting because of the uncertainty in the nature of occurrences of these events.
2.2 Some Basic Rules of Probability

- For any event $A$, $0 \leq P(A) \leq 1$ (i.e. Probabilities lie between 0 and 1).
- Zero probability implies that the event is unlikely to occur or just impossible.
- A probability of 1 implies that the event is certain.
- The compliment of an event $A$ (i.e. NOT $A$) is the event that happens exactly when $A$ does not occur. $P(\overline{A}) \geq 1 - P(A)$
- Conditional probability is the probability of an event occurring given that another event has occurred.

3.0 Differences between Deterministic and Probabilistic Forecasts

Forecast can be presented either as deterministic or probabilistic. Short-term forecasts (Nowcasts) are almost entirely deterministic in that they state exactly what is going to happen, when and where. Examples of deterministic forecasts may be given by statements such as:

(i) Rainfall will be above average this season
(ii) Rainfall will be 50% above average this season.
(iii) The afternoon temperature in a given city will be 28ºC.

Probabilistic forecasts on the other hand are forecasts that give the probability of an event of a certain (range of) magnitudes may occur in a specific region in a particular time period. An example of probabilistic forecast would be like there is a 70% chance that rainfall will be above average in the coming season. This implies that in many past occasions (using historical information), the calculations has led to an estimate of 70% probability of the observed rainfall actually being above average in about 70 out of every 100 events.

In general, longer times scale forecasts such as seasonal to interannual are mostly probabilistic and the forecasts are given in probabilistic ranges for the season. It should be noted that it is possible to convert probabilistic forecasts into deterministic forecasts.
4.0 Seasonal to Inter-annual climate forecasting

Weather forecasts provide information about the weather expected over the next few days. While it is generally not possible to predict these day-to-day changes in detail beyond about a week ahead (Washington and Downing, 1999), it is possible to say something about the likely conditions averaged over the next few months. Seasonal forecasting is an attempt to provide information on the likely conditions of the weather several months or more into the future.

The current scientific approach behind seasonal forecasting relies on the fact that lower-boundary forcing, which gives rise to atmospheric perturbations, evolves more slowly over the course of a season than the atmospheric perturbations themselves and that the response of the atmosphere to this forcing is predictable (Sarah Murphy et al., 2001). The feasibility of seasonal forecasting depends on the fact that over a season, the effects of short-term (weather) events tend to average out, revealing the smaller but more consistent influence of the ocean SST anomalies (Carson, 1998) and land surface on the atmosphere, (Nicholson, 1988). It should be noted that at the seasonal timescale, detailed forecasts of weather events or sequences of weather patterns are not possible.

4.1 Approaches to Developing Probabilistic Seasonal Forecasts

Seasonal climate forecasting procedures normally start with historical climate records, or a climatological database. This database (Temperature or precipitation) should be 'clean' (quality controlled), complete and as long as is possible. A standard 30-year period, such as 1961-90 or longer is applied for most databases. Various statistical analyses can then be carried out on the historical data and some relationships determined between potential predictors (e.g., ENSO and associated teleconnections) and the predictand (Temperature or precipitation). The common predictor choices are usually lagged SST anomalies over the global oceans that are considered pertinent to the predictand.
Performing simple correlation analysis between the predictand and predictor variables is among the statistical techniques being used in identifying the lagged statistical associations between SST indices and climate variables. Once these have been established, what follows is to develop either simple linear regression or multiple linear regression (deterministic) models relating the predictors with the predictands and use the model for prediction. Several other potential predictors such as QBO, SOI and SST gradients among others can be included in the development of the regression model. Stepwise regression technique (forward/backward) can be used in selecting the best predictors that are to be included in the multiple linear regression equation.

A simple linear regression takes the form:  
\[ Y = a_0 + a_1 X + \epsilon \]. This is a probabilistic linear relationship of \( Y \) (Predictand) and \( X \) (Predictor). The functional relationship contains a deterministic part, \( a_0 + a_1 X \) and a random error component term, \( \epsilon \). The regression constants \( a_0 \) and \( a_1 \) can be determined from the sample data using Least Squares Method.

In the fitting of a multiple regression model, a single predictand, \( Y \), has more than one predictor variable, \( X \). If \( k \) is the number of predictor variables in the model, then the multiple linear prediction equation may be given by: 
\[ Y = b_0 + \sum_{i=1}^{k} b_i x_i + \epsilon_i \], where \( b_0 \) and \( b_i \) are the intercept and regression coefficients for the predictors, \( x_i \). The variance of the error term \( \epsilon \) in this case can be computed using the sample data and is given by 
\[ S^2 = \frac{SSE}{n-(k+1)} \]. The model adequacy can be tested through computing the multiple coefficient of determination (\( R^2 \)) given by 
\[ R^2 = 1 - \frac{SSE}{\sum_{i=1}^{n} (Y - \bar{Y})^2} \]. For \( R^2 = 0 \), it implies Lack of fit, while \( R^2 = 1 \) implies perfect fit.

Examples of linear regression models developed for some selected rainfall stations in Kenya for the MAM season are shown in figures 1 - 3. The model training period for these cases were from 1961 to 1990 while the remainder of the period was used for model verification. The various model equations and their respective fits are also indicated.
Z7 = -0.04 + 0.30*NPA2M1 - 0.68*SPA1M1 - 0.49*SPA4M1 - 0.53*ZAFM11 \hspace{1cm} R^2 = 64% 

**Figure 1:** Comparison between Observed and Forecasted MAM rainfall anomalies for Voi station in Kenya

Z8 = 0.04 - 0.46*SIN3M1 + 0.83*NPA1M12 - 0.59*NPA1M1 + 0.37*MIB3M1 \hspace{1cm} R^2 = 61% 

**Figure 2:** Comparison between Observed and Forecasted MAM rainfall anomalies for Kerugoya station in Kenya
Figure 3: Comparison between Observed and Forecasted MAM rainfall anomalies for Garbatula station in Kenya

5.0 Verification of Forecast Skills

A number of quantities can be computed as a means of verifying forecast skills. The different verification measures may be any of the following:

Accuracy: This is a general term indicating the level of agreement between the forecasted value and the true observed value. The difference between the two values is the error. The smaller the error the greater the accuracy.

Skill: This measures the accuracy of a given forecast relative to the accuracy of forecasts produced by some standard procedure. Skill scores provide a means of accounting for variations in accuracy that have nothing to do with the forecaster's ability to forecast.

Reliability or bias: This may be defined as the average agreement between the stated forecast value of an element and the observed value. Mathematically,

\[ BIAS = \frac{1}{N} \sum_{i=1}^{N} (F_i - O_i)^2 \]  

A positive bias indicates that the forecast value exceeds the
observed value on the average, while a negative bias corresponds to under forecasting the observed value on average.

**Resolution** is the ability of the forecast to sort or resolve the set of sample events into subsets with different frequency distributions. It measures the state of art and is tied to the overall experience and understanding of the forecaster.

**Sharpness** is the tendency to forecast extreme values

5.1 **Use of Contingency Tables and Some of the Associated Scores**

A useful summary of the forecast and observed climate events can be represented in the form of contingency tables. These tables provide the basis from which a number of useful scores can be obtained.

<table>
<thead>
<tr>
<th>Forecast category</th>
<th>Above-Normal</th>
<th>Near-Normal</th>
<th>Below-Normal</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above-Normal</td>
<td>A11</td>
<td>A12</td>
<td>A13</td>
<td>J</td>
</tr>
<tr>
<td>Near-Normal</td>
<td>A21</td>
<td>A22</td>
<td>A23</td>
<td>K</td>
</tr>
<tr>
<td>Below-Normal</td>
<td>A31</td>
<td>A32</td>
<td>A33</td>
<td>L</td>
</tr>
<tr>
<td>TOTAL</td>
<td>M</td>
<td>N</td>
<td>O</td>
<td>T</td>
</tr>
</tbody>
</table>

**Percent Correct**

\[ \text{Percent Correct} = \frac{A_{11} + A_{22} + A_{33}}{T} \times 100 \]

**Post Agreement, False Alarm Ratio (FAR)**

- Post Agreement is the number correct forecasts made divided by the number of forecasts for each category
- Post Agreement A11/M, A22/N, A33/O for the three categories
- False Alarm Ratio (FAR) is sensitive only to false predictions of the severe events, not to missed events.
- FAR = 1 - Post Agreement of the severe event
Probability of Detection (POD)

This is the number correct divided by the number observed in each category. It is a measure of the ability to correctly forecast a certain category, and is sometimes referred to as "Hit rate" especially when applied to severe weather verification.

\[ \text{POD} = \frac{A_{11}}{j}, \frac{A_{22}}{k}, \frac{A_{33}}{l} \text{ for the three different categories} \]

Bias

Bias is the number of forecast divided by the number observed for each category. It measures the ability to forecast events at the same frequency as found in the sample without regard to forecast accuracy.

- Bias = \( \frac{M}{J}, \frac{N}{K}, \frac{O}{L} \) for the three categories, where Bias = 1 implies no Bias.
- Bias >1 implies over-forecasting the event
- Bias <1 implies under-forecasting the events

Critical Success Index (CSI)

\[ \text{CSI} = \frac{A_{11}}{M + J - A_{11}} : \frac{A_{22}}{N + K - A_{22}} : \frac{A_{33}}{O + L - A_{33}} \]

Skill Score (Heidke Skill Score)

\[ SS = \frac{R - E}{T - E} \]

where R = number of correct forecasts,
T = total number of forecasts,
E = number expected to be correct based on chance, persistence, climatology, etc.

\[ HSS = \frac{(A_{11} + A_{22} + A_{33})}{T} \cdot \frac{JM + KN + LO}{T} \]

The term "Heidke" skill score in particular is most often associated with chance as the standard of comparison and is a popular verification statistic.

Linear Error in Probability Space (LEPS)

LEPS operates similar to the Hits skill Score, but penalises a forecast that is 2 categories in error more than a forecast that is one category in error.
Table 1 below shows an example of a contingency table for the model output skill scores for zone 8 represented by Kerugoya station. Skill score outputs for other locations can similarly be generated. The outputs from such tables can then be used in issuing seasonal forecasts by way of tercile method presented in the next section.

<table>
<thead>
<tr>
<th>Zone 8 (Kerugoya) Forecasts</th>
<th>Dry</th>
<th>Normal</th>
<th>Wet</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBS.</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Dry</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Normal</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Wet</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

Percent Correct = 64%

<table>
<thead>
<tr>
<th>Probability of Detection</th>
<th>Dry</th>
<th>Normal</th>
<th>Wet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post Agreement</td>
<td>100</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>False Alarm (1st Order)</td>
<td>60</td>
<td>67</td>
<td>67</td>
</tr>
</tbody>
</table>

Hit Skill score (HSS) = 0.46

5.1 Presentation of Forecast Outlooks using terciles

Seasonal climate outlooks are usually presented in a slightly different format compared to daily forecasts. A standard format for presenting these forecast is by assigning percentage probabilities into what are known as terciles. Terciles basically consist of three ranges of values that are used to represent three broad sectors of a normal probability distribution with equal chances of occurrence, climatologically, namely the lower, middle, and upper thirds of the expected distribution of values. For a 30-year record, each tercile would cover a 10 year period. In a typical year, there is equal probability that rainfall will fall into the above-, near-, and below-normal categories (33.3% chance for each category. This equal probability distribution is referred to as “climatology”.

For example, a typical seasonal forecast may be presented as (45, 30, 25) which can be interpreted as a 45% chance of seasonal total precipitation being in the upper (Wet) tercile, a 30% chance of it being in the middle or (Near normal) category, and a 25% chance that it may fall in the lowest third (Dry) category.
Seasonal forecasts presented in this way not only indicate the most likely outcome for the upcoming months or seasons, but also the distribution of possible outcomes. Such forecasts can take all the possible directions and can therefore never be wrong.

The use of analogue techniques is another important tool for forecasting. In this case, comparison is made between the season or year under investigation with past or historical data and any similarities noted. Detailed analysis can be made on the selected analogues years to determine the best analogue data to be used in providing climate information to users.

Table 2 below presents an example of ranked rainfall anomalies grouped into Terciles for some rainfall zones in Kenya. Figure 4 shows an example of the presentation of seasonal forecasts using terciles.

<table>
<thead>
<tr>
<th>Year</th>
<th>Zone1</th>
<th>Year</th>
<th>Zone2</th>
<th>Year</th>
<th>Zone3</th>
<th>Year</th>
<th>Zone4</th>
<th>Year</th>
<th>Zone5</th>
<th>Year</th>
<th>Zone6</th>
<th>Year</th>
<th>Zone7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>-0.11</td>
<td>1965</td>
<td>-1.33</td>
<td>1953</td>
<td>-1.06</td>
<td>1984</td>
<td>-1.17</td>
<td>1974</td>
<td>-1.53</td>
<td>1971</td>
<td>-1.20</td>
<td>1995</td>
<td>-1.06</td>
</tr>
<tr>
<td>1990</td>
<td>-0.99</td>
<td>1994</td>
<td>-1.05</td>
<td>1976</td>
<td>-0.18</td>
<td>1983</td>
<td>-0.46</td>
<td>1969</td>
<td>-1.31</td>
<td>1974</td>
<td>-1.19</td>
<td>1985</td>
<td>-1.05</td>
</tr>
<tr>
<td>1995</td>
<td>-0.99</td>
<td>1998</td>
<td>-1.02</td>
<td>1999</td>
<td>-0.83</td>
<td>1992</td>
<td>-0.96</td>
<td>1965</td>
<td>-1.28</td>
<td>1965</td>
<td>-0.82</td>
<td>2001</td>
<td>-0.99</td>
</tr>
<tr>
<td>1998</td>
<td>-0.98</td>
<td>1979</td>
<td>-0.70</td>
<td>1994</td>
<td>-0.83</td>
<td>2000</td>
<td>-0.92</td>
<td>1977</td>
<td>-1.14</td>
<td>1964</td>
<td>-0.80</td>
<td>1985</td>
<td>-0.79</td>
</tr>
<tr>
<td>1976</td>
<td>-0.95</td>
<td>1995</td>
<td>-0.68</td>
<td>1976</td>
<td>-0.44</td>
<td>1992</td>
<td>-1.09</td>
<td>1976</td>
<td>-0.79</td>
<td>1976</td>
<td>-0.79</td>
<td>1990</td>
<td>-1.00</td>
</tr>
<tr>
<td>1990</td>
<td>-0.99</td>
<td>1998</td>
<td>-1.02</td>
<td>1999</td>
<td>-1.00</td>
<td>1999</td>
<td>-0.96</td>
<td>1994</td>
<td>-1.05</td>
<td>1993</td>
<td>-1.05</td>
<td>1993</td>
<td>-1.00</td>
</tr>
<tr>
<td>1995</td>
<td>-0.99</td>
<td>1998</td>
<td>-1.00</td>
<td>1999</td>
<td>-1.00</td>
<td>1999</td>
<td>-0.89</td>
<td>1994</td>
<td>-1.05</td>
<td>1993</td>
<td>-1.00</td>
<td>1993</td>
<td>-1.02</td>
</tr>
<tr>
<td>1998</td>
<td>-0.98</td>
<td>1979</td>
<td>-0.70</td>
<td>1994</td>
<td>-0.83</td>
<td>2000</td>
<td>-0.92</td>
<td>1977</td>
<td>-1.14</td>
<td>1964</td>
<td>-0.80</td>
<td>1985</td>
<td>-0.79</td>
</tr>
</tbody>
</table>

Table 2: Tercile groupings for some rainfall zones in Kenya.
Figure 4 presents an example of the seasonal rainfall forecasts for the MAM 2002 rainfall in Kenya.

For this particular MAM forecast, which is the main rainy season in Kenya, it can be seen that much of the country has an enhanced probability of experiencing near to above normal rainfall. The dissemination of such forecasts is communicated to a broader group of potential users through organizing Climate Outlook Fora.
6.0 Summary and Conclusions

The current scientific approaches to seasonal forecasting have been discussed and a distinction between deterministic and probabilistic forecasts presented. Some useful ideas on probability and statistics have also been presented including the basic stages in developing probabilistic forecasts using empirical statistical methods. The methodologies for computing various forecast skill score are given including the generation of probability tercile forecasts. Finally, it has been shown that probabilistic forecasts can offer valuable and information for decision making than deterministic ones.

7.0 References


Sarah J. Murphy, Richard Washington1, Thomas E., Downing, Randall V. Martin, Gina Ziervogel, Anthony, Preston1, Martin Todd, Ruth Butterfield And Jim Briden, 2001 Seasonal Forecasting for Climate Hazards: Prospects and Responses *Natural Hazards* 23: 171–196, 2001