Signal To Noise Ratio Applied to COADS Ship-Measured Variables

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Objective

• Develop a method to use the sampling error of monthly box averaged ship data in a meaningful way

• Should be a function of ship location in time and space

• Method should be flexible enough to accept simple averages or OI

• Must be practical in application
I-COADS Time-Space Box

Divide Box into ‘m’ sub-boxes

Each sub-box has at most 1 ship observation

‘n’ total number of ships per box
**Signal To Noise Ratio**

\[
S/N = \text{Month-to-Month Variance/ Error Variance}
\]

**Sampling Error or Error Variance:**

\[
s_e^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} (\beta_i - \mu)^2 + \sum_{i=1}^{m} \sum_{j=1}^{n} (\gamma_j - \mu)^2
\]

Assume \( \sigma^2_p = E[(R(t) - \mu)^2] \) unbiased

\( \rho = \text{time-space correlation function of vector distance ‘d’} \)

1. Find expression for $\sigma^2_T(n) = E[(R_t(n) - \mu)^2]$

   - where $R_t(n)$ is the sample ave
   - $\mu$ is the long-term mean

2. Let $n = \infty$

3. Then compute $S/ N = \sigma^2_T(\infty) / \sigma^2_e(n)$
Equation for $\sigma_T^2(n)$

$$S_T^2 = S_p^2$$

$$S_T^2 = S_p^2 VR$$

1. $E[\rho]$?

2. $VR$?
What is $\rho$ and $E[\rho]$?

- $\rho$ is the time-space correlation function.
- $E[\rho]$ is the expected value of the correlation $\rho(x,y,t)$.
- Difficult to find the $\rho(x, y, t)$ function easily.
Practical Way to Compute the $\rho$ Function

• Assume $\rho(t)$ is independent of $\rho(x, y)$
• Thus, $\rho(x, y, t) = \rho(x, y) \rho(t)$
• The correlation function is separable in time and space?

(Rodriguez-Iturbe and Mejia, 1974, WWR)
\[ E[\rho] = \int \int \rho(x, y, t) f(L) dL \]

**Ave Domain**

\[ L = \text{Vector time-space distance between two ship locations} \]

\[ f(L) = \text{pdf of vector time-space distances between two ship randomly located locations} \]
Variance Reduction Factor VR

As VR decreases with increasing observations, the monthly variance estimated from n observations approaches the true monthly variance.

\[ \sigma_T^2 = \sigma_p^2 \left( \frac{1}{n} + \frac{(n-1)E[\rho]}{n} \right) \]

\[ = \sigma_p^2 \cdot VR \]

“n = number of ships”

Take Limit \( n \to \infty \), VR \( \to E[\rho] \)

Thus, \( \sigma_T^2(\infty) = \sigma_p^2 \cdot E[\rho] \)
True Monthly Variance $\sigma_T^2$

Thus, by setting the number of Observations = infinity
We obtain the true monthly variance $\sigma_T^2$ for a given monthly box

Signal to Noise Ratio:

\[
= \frac{\sigma_T^2(\infty)}{\sigma_e^2(n)}
\]
I-COADS Analysis

- Ship Air Temperature observations from sample month
- Location limits: 120E to 160E and 20S to 20N
- Only those boxes with sufficient observations included in analysis
Study Locations (and # of obs 1950-1997)
Signal To Noise Ratio (sample month)
Conclusions

• Method allows a S/N ratio value to be placed on every I-COADS monthly box
• Accounts for month-to-month ship location variation in time and space
• May be applied to OI schemes
• More work needed on the validity of statistical assumptions