Generalized Wave-Ray Approach for Propagation on a Sphere and Its Application to Swell Prediction

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Outline of Talk

- Introduction
- Wave-Ray Approach
- Line Source for Swell Propagation
- Model Comparisons
  - Simplified Test Cases
  - Pacific Ocean Sea States
- Conclusions
- A Diversion: Model Source Terms
Motivations for the Work

- Improved form for incorporating depth refraction in wave models
- Investigations of swell propagation and swell decay
Refraction Processes

- Not generally important at oceanic scale, but a consideration with nested sub-grids
Lake Hindcasts

Bathymetry of Lake Erie and Lake Saint Clair

Source: US Geophysical Data Center, NOAA
Model Development

- Context of 2G models where wave energy propagation consumes at least 50% of the computation time – less important for a 3G model

- Appeal of a 2G Model:
  - *Computational speed (25x to 40x factor)*
  - *Scalability (any size waterbody)*
Basic Development

- Method of characteristics approach following great circle routes and depth-induced curvature (ray back-tracking)

$E(\lambda_1, \phi_1, \theta, f, t+\Delta t)$

$E(\lambda_2, \phi_2, \theta, f, t)$

Interpolation Approach?
Basic Development

Solve for intersection location and angle by iteration

\[ E(\lambda_1, \phi_1, \theta, f, t+\Delta t) \]

\[ E(\lambda_2, \phi_2, \theta, ft) \]
\[ \hat{E}(f, \theta)_{\text{intersection}} = \lambda_{11} \hat{E}_{i\pm n, j\pm m}(f, \theta_1) + \lambda_{12} \hat{E}_{i\pm n', j\pm m'}(f, \theta_1) + \lambda_{21} \hat{E}_{i\pm n, j\pm m}(f, \theta_2) + \lambda_{22} \hat{E}_{i\pm n', j\pm m'}(f, \theta_2) \]

\[ \lambda_t = \frac{c_g \Delta t}{D} \]

\[ \hat{E}_{i,j}(f, \theta)^{(n)} = (1 - \lambda_t) \hat{E}_{i,j}(f, \theta)^{(n-1)} + \lambda_t \hat{E}(f, \theta)_{\text{intersection}} \]
Extension to Higher Order

- Higher order interpolation – cubic spline, polynomial, ....
- In the end we utilized a relatively simple procedure of determining the great circle arc intersections with adjacent grid cells, and using cubic polynomial interpolation
Higher Order – Cubic Polynomial Interpolation

\[ E(\lambda_1, \phi_1, \theta, f, t + \Delta t) \]

\[ E(\lambda_2, \phi_2, \theta, f, t) \]
Gaussian Wave Burst
Extension to Simulation of Swell Propagation

- Relate waves arriving at any point on the earth to waves passing a fixed, non-intersecting line (a “line source”)

\[ E(x_0, y_0, f, \theta, t) = E(x, y, f, \theta(x, y, x_0, y_0), t - D/c_g) \]

- Fundamental assumption: source terms negligible over the propagation distance
Line Source

- Integration of wave information along a given line
- Low-frequency (< 0.1 Hz) spectral information stored at regular intervals & replaces the computed wave energy densities in relevant directional and frequency bins
Lines of Swell Wave Propagation Initiated at 30 deg N Latitude
Tp = 20 s

Note: circles on each line are 24 hours apart

11,100 km
43 deg angle
30 deg

Store spectral information along a line of latitude
Line Source

- Low-frequency energy propagated without diffusion
- Effects of islands and sub-grid blocking can be readily incorporated
The Wave Model

- Propagation schemes implemented into a 2\textsuperscript{nd} Generation wave model WAVAD
- Some comparisons performed with Wavewatch III v2.22
  - *Settings as per NCEP Operational settings (Tolman & Chalikov, Quickest/Ultimate & Tolman Averaging)*
- Model grid 1°x 1°; 24 directional bins; 24 frequencies; sub-grid blocking represented
Gaussian Burst of Wave Energy (No Source)
Comparative Results at 80°W, 30°S

Wave Height Comparisons at 80 deg W, 30 deg S for Gaussian Waves Initiated at 165degW, 45degN

- WAVAD - First Order
- WAVAD - Higher Order
- WW3 - First Order
- WW3 - Quickest
- Line Integral

Storm Initiation
Selection of Different Line Source Locations for Gaussian Burst at 45 deg N

Wave Height Comparisons at 80 deg W, 30 deg S for Gaussian Waves Initiated at 165 deg W, 45 deg N

- Line Source at 40 deg N
- Line Source at 30 deg N
- Line Source at 20 deg N
Theoretical Moving Cyclonic Storm

Target location
Comparative Results at 80°W, 30°S

Wave Height Comparisons at 80deg W, 30deg S for Synthetic ETC

- WAVAD - First Order
- WAVAD - Higher Order
- Line Integral

Time (days): 1 6 11 16 21 26 31
Hm0 (m): 0 0.5 1 1.5 2 2.5 3

Storm Initiation
Simulation of More Realistic Sea States for Comparative Purposes

- NCEP/NCAR Re-Analysis Winds
- Only simplistic modification to winds in equatorial region based on buoy data & scatterometer (linear factor)
- One year – 2001
Use of higher order wave-ray scheme had no significant impact on results.

Note: sheltering issues, refraction.
Note: sheltering issues
Typical Wave Height Comparison with Line Integral

Lagrangian Approach
WAVAD Propagation Scheme
Recorded Wave Data

Baird
Swell Predictions from a Buoy Network

Can't predict swell from a single buoy
Conclusions

- An outline of semi-Lagrangian wave-ray approach to wave propagation has been shown with extensions to higher order and prediction of swell from a line source.
- Line source approach didn’t yield overall improvements under realistic sea states in the Pacific Ocean
  - May aid in defining specific swell events
  - Useful method for isolating source term effects and evaluating diffusion in propagation schemes
  - Can assess areas of “influence” when assimilating buoy data
A Diversion……..

With discussion of Boltzman integrals, etc. coming up over the next couple of days…….

\[
\frac{\partial n_1}{\partial t} = \iiint G(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \times \delta (\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \times \delta (\omega_1 + \omega_2 - \omega_3 - \omega_4) \\
\times \left[n_1 n_2 (n_3 + n_4) - (n_1 + n_2) n_3 n_4\right] \,dk_2 \,dk_3 \,dk_4
\]

Yikes!
Source Term Comparisons

- Observations when carrying out model comparisons
  - WAVAD
  - WaveWatchIII (v2.22, operational parameters)
Comparisons to Pierson-Moskowitz Limit

![Graph showing comparisons between WW3 and PM Fully-Developed Limit]

- **X-axis**: Wind Speed (m/s)
- **Y-axis**: $H_m0$ (m)

Legend:
- **Blue Line**: WW3
- **Pink Line**: PM Fully-Developed Limit
Rate of Development

Comparison of Wave Growth

U=12 m/s

See Rogers (2002)
Drag Coefficient

- WW3
- WAVAD - Garratt (1977)
Observations Regarding Source Terms

- Wide difference in behaviour exhibited
- Implications towards our understanding of wave physics?
- Interesting that similar results can be provided under such disparate source behaviour
- Can we understand swell decay with the existing buoy network?
- Have we learned much since Snodgrass et al. (1966)?