Using adjoint sensitivity to control discretization errors: Goal-oriented adaptivity for idealized tropical cyclone scenarios

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Motivation

- Forecasting development and motion of TCs: severe challenge for NWP
- Interacting processes on a large range of scales (< 1km … 10,000km)
- Multiscale modelling necessary, but expensive → grid adaptation
- Common approach: Grid nesting (how deep? may miss sensitive regions…)
- Goal-oriented adaptivity: promising approach for automatic grid adaptation
Goal-oriented Error Estimation

**Idea:** Given a quantity of interest (goal functional) $J$, adapt the grid in a way that minimizes the error in $J$ (e.g. under the constraint that the computational effort is constant).

**Realisation:** For each cell, estimate contribution to error in $J$ using an a posteriori error estimator. Increase resolution where error is large.

A posteriori error estimator: $|J(u) - J(u_h)| \approx E(u_h) \leq \sum_{K \in T_h} \eta_K$ with $\eta_K \geq 0$

Start from coarse grid, iterate until total error sufficiently small...
**Linear sensitivity analysis**

- Adaptation requires information about future impact of discretization errors

Assume \( J(y) \) depends on state \( y = u(t_2) \)

Impact of discretization errors at \( t_1 (< t_2) \):

\[
\delta J = \langle \partial_y J, \delta y \rangle = \langle \partial_y J, L \delta x \rangle = \langle L^T \partial_y J, \delta x \rangle
\]

adjoint sensitivity = dual solution

Discretization error can be estimated in finite element codes: higher order – lower order solution

with \( x = u(t_1) \), tangent linear model \( L \) and adjoint model \( L^T \)

→ A well-known but often hard to interpret quantity measuring sensitivity is generated during the grid adaptation process.

- Adjoint sensitivity can also be interpreted as optimal perturbation
- Idealised models: Clearer view on selected perturbation growth processes, better suited to test automatic grid adaptation than full-physics 3D models
Test case: TC-TC Interaction

- Influences motion (Fujiwara) and structure (shear)
- Occurs ~2-3 times per year, often associated with low predictability
- Idealised, **nondivergent-barotropic** case: TC-like vortices on f-plane
- **High sensitivity** to initial TC separation (extreme case: bifurcation between merger and non-merger cases) and numerical errors

Vorticity evolution for initial TC separations 370km (top) and 400km (bottom) in the first 48 hours.
Adaptive Simulation

Goal functional $J = \int_{\text{core of left vortex}} \zeta(96h, x, y) \, dx \, dy$, static grid
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Vorticity

Dual velocity
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Adaptive Simulation

Goal functional $J = \int_{\text{core of left vortex}} \zeta(96h, x, y) \, dx \, dy$, static grid
• Adaptation → Reduction of error in goal functional and cyclone position

• #DoFs required to get correct type of solution (non-merging):
  20,000 on adaptive grid, 80,000 on uniform grid
Adaptation in time

Temporal error indicators

Position error [km]

- Error indicators allow for an optimization of the time partitioning
- Reduction of position error by one order of magnitude

**goal-oriented adaptation → highly efficient meshes in space and time**
Dual solution vs. Adjoint sensitivity vs. Singular Vectors

\[ J = \int_{|v|>0.9 \max(|v|)} |v|^2 \, dA \]

\[ J = \int_{\zeta>0.5 \max(\zeta)} \zeta \, dA \]

Leading singular vector (energy norm)
Initial and Evolved Optimal Perturbations

**Initial perturbation \((L^T \partial_y J)\)**

Elongated structures aligned with separatrices in corotating frame

**Evolved perturbation \((LL^T \partial_y J)\)**

Vorticity dipoles → perturbations cause displacement of the TCs

Initial perturbation grows by Orr effects (untilting & unshielding) and accumulates near stagnation points, associated velocity field causes a displacement of the vortices.
Initial sensitivity structure

interacting vortices

vortices in horizontal shear

+ rotation
(transforming into corotating frame = subtraction of vorticity → anticyclones)

see Poster 55394 - “Singular vectors for idealised TC scenarios: Structure and perturbation growth mechanisms”
TC-Interaction in 3D

- Vertical shear caused by first vortex tilts second (and vice versa)
- Tilted vortices precess and modify TC motion → not only initial separation, but also stability and strength of coriolis force determine the outcome of the interaction
- Abrupt changes in TC motion at certain precession angles

Vorticity evolution for the same initial TC separations but f-values corresponding to different latitudes.
Dual solution for 3D case

Work in progress...

First results:

- 3D dual vorticity is aligned with 2D separatrix near ground
- Change of sign with height indicates that not only displacement but also tilting of the vortices plays a role

View from SW / above

View from top
Conclusion

• Goal-oriented adaptivity: automatic grid adaptation in space and time for multiscale problems like tropical cyclone forecasts → highly efficient grids
• Ability to detect remote sensitive regions could justify increased effort for the grid adaptation, compared to simpler refinement approaches
• Generated during grid adaptation: dual solution = adjoint sensitivity
• Idealised scenarios: Interpretation of sensitivity structure and understanding perturbation growth mechanisms is much easier
• Flow boundaries are preferred location for perturbation growth in 2D → could also be important in full-physics 3D case

Publications
• Bauer, Baumann, Scheck, Gassmann, Heuveline, Jones: Simulation of tropical-cyclone-like vortices in shallow-water ICON-hex using goal-oriented r-adaptivity, 2014, TCFD, 28, 1, 107-128
• Baumann, Heuveline, Scheck & Jones: Goal-Oriented Adaptivity for Idealised Tropical Cyclones: A Binary Interaction Scenario, submitted to Meteorologische Zeitschrift