Efficient Radar Forward Operator for Operational Data Assimilation and Model verification within the COSMO-model

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Project goals

- Develop efficient radar forward operator suitable for:
  - Ensemble based radar data assimilation LETKF
  - Model microphysics verification

- Comprehensive code in a modular fashion, and simulation of each physical process with different levels of complexity (state-of-the-art methods), which enables to determine the setup with the "best" compromise between efficiency and accuracy for different applications.

- Online coupling to the COSMO-model, parallelized and vectorized code, easy framework to use

- Honor all COSMO-model microphysical schemes
Advantages of radar data

- very high temporal (e.g., 5 min) and spatial resolutions (convective scale)
- full coverage of radar network over Germany
- valuable data source for model verification and data assimilation

Figure 1: German network of 17 dual Doppler C-band radars
Why is radar forward operator required?

Figure 2: Radar vs Model

Radar volume scan, coordinates: radar system \((r_0, \alpha_0, \epsilon_0)\) or beam system \((r_0, \phi, \theta)\), observables: reflectivity \(Z_e\), Doppler velocity \(v_r\); COSMO-model, coordinates: \((i, j, k)\), model variables: wind vector \((u, v, w)\), temperature \(T\), total pressure \(p\), partial pressure \(e\)

Conclusion: radar scan geometry \(\neq\) model grid and radar observations \(\neq\) model variables, so they are not directly comparable. What we can do is

\[ Z_e, v_r \Rightarrow (u, v, w), T, p \text{ or } e: \text{complicated and the mapping not unique, or} \]
\[ (u, v, w), T, p, e \Rightarrow Z_e \text{ or } v_r: \text{much more convenient, this transformation is called radar forward operator} \]
Radar forward operator

In words, the radar forward operator simulates the physical processes involved in radar measurement and radar observables from the prognostic model outputs and allows for a direct comparison in radar observational space.
What to simulate?

Figure 3: Radar measurement process: $\ell$ attenuation factor, $f^2$ beam weighting function, $\sigma_b$ backscattering cross section, $\sigma_{\text{ext}}$ extinction cross section, particle distribution function $N$, $\Lambda$ extinction coefficient

Thus, simulation includes:

1. Beam propagation effects like beam bending and broadening
2. Reflectivity, Doppler velocity and attenuation
How to simulate radar beam bending

Methods:

- **43ERM**: 4/3 Earth Radius Model:
  - offline = constant refractivity gradient (-40 N-units km\(^{-1}\));
  - straight-line propagation, easy implementation and parallelization

- **SODE**: Second-order Ordinary Differential Equation:
  \[
  \frac{d^2 h}{dr^2} + \left( \frac{dh}{dr} \right)^2 \left( \frac{1}{n \, dh} + \frac{1}{R_E + h} \right) - \left( \frac{1}{n \, dh} + \frac{1}{R_E + h} \right) = 0
  \]
  - online = consider real-time refractivity;
  - sophisticated parallelization required

Figure 4: Beam propagation depending on gradients of refractive index N and modified refractive index M

Tests on SODE with an idealized ducting case with the COSMO-model:

Figure 5: Tests with different initial elevations (\(\epsilon_0 = 0.1^\circ\) and \(0.5^\circ\)) and namelist parameters exp_galchen influences the vertical resolution of model. The larger exp_galchen is, the denser the lower vertical levels are. For \(\epsilon_0 = 0.1^\circ\) and exp_galchen = 3.6, a ducting propagation is detected.

Paper "Radar Beam Tracing Methods Based on Atmospheric Refractive Index" minor revision at JAOTECH, 2014
How to simulate radar beam broadening

Figure 6:
Several subrays are used to represent one single beam. The initial elevation and azimuth of subrays are determined by the numerical integration method "Gauss-Legendre quadrature". The values on subrays are then collected and averaged to get the representative reflectivity and Doppler velocity for this pulse volume.

Test with mountain set in the northeast:

Figure 7:
PPI (at elevation 0.5°) of \( v_r \)
How to simulate (attenuated) reflectivity

Assumption: $Z_e$ and $\Lambda$ do not vary much in radial distance within pulse volume, so only two-dimensional integration

$$
\langle Z_e^{(R)}(\vec{r}_0) \rangle = \frac{\int_{\alpha_0 - \pi}^{\alpha_0 + \pi} \int_{\epsilon_0 - \pi/2}^{\epsilon_0 + \pi/2} Z_e \exp \left( -2 \int_{0}^{r} \int_{0}^{\infty} \sigma_{\text{ext}} N dD \right) f^2 \cos \epsilon \, d\epsilon \, d\alpha}{\int_{\alpha_0 - \pi}^{\alpha_0 + \pi} \int_{\epsilon_0 - \pi/2}^{\epsilon_0 + \pi/2} f^2 \cos \epsilon \, d\epsilon \, d\alpha}
$$

Simplification examples: (via namelist parameters)

- No attenuation: $\langle Z_e^{(R)}(\vec{r}_0) \rangle = \frac{\int_{\alpha_0 - \pi}^{\alpha_0 + \pi} \int_{\epsilon_0 - \pi/2}^{\epsilon_0 + \pi/2} Z_e f^2 \cos \epsilon \, d\epsilon \, d\alpha}{\int_{\alpha_0 - \pi}^{\alpha_0 + \pi} \int_{\epsilon_0 - \pi/2}^{\epsilon_0 + \pi/2} f^2 \cos \epsilon \, d\epsilon \, d\alpha}$

- No beam smoothing: $\langle Z_e^{(R)}(\vec{r}_0) \rangle = Z_e \ell^{-2}$
Approximations for $Z_e$ and $\Lambda$

$$Z_e \sim \int_0^{\infty} \sigma_b(D, m)N(D)dD \quad \Lambda = \int_0^{\infty} \sigma_{\text{ext}}(D, m)N(D)dD$$

where $m$ refractive index of hydrometeor.

2 major options in operator:

1. **Mie scattering** for $Z_e$ and $\Lambda$:
   - Hydrometeors treated as simple spheres (rain; dry and melting graupel; dry hail) or two-layered spheres (dry and melting snow; melting hail)
   - Sub-options for $m$ of dry or melting solid particles: $\approx 20$ flavours of Maxwell-Garnett- and Bruggemann effective medium approximations (EMAs)
   - Numerical integration of the $Z_e$ and $\Lambda$ integrals (Look-up tables for speed up)

2. **Rayleigh approximation** for $Z_e$ ($\Lambda = 0$ in this case):
   - $\sigma_b = \frac{\pi^5 |K|^2}{\lambda^4} D^6$, $K = \frac{m^2-1}{m^2+2}$
   - For dry or melting solid particles, $m$ from simpler EMA (Oguchi, 1983)
   - $\implies$ Efficient analytic calculation of $Z_e$ for each hydrometeor species

Documentation "RADAR_MIE_LM and RADAR_MIELIB — Calculation of Radar Reflectivity from Model Output", available on request
How to simulate Doppler velocity

\[

v_r({\vec{r}_0}) = \frac{\alpha_0 + \pi}{\alpha_0 - \pi} \frac{e_0 + \pi/2}{e_0 - \pi/2} \int_{\alpha_0}^{\alpha_0 + \pi} \int_{\epsilon_0}^{\epsilon_0 + \pi/2} (\vec{v} \cdot \vec{e}_r) \frac{Z e f^2 \cos \epsilon \delta \alpha}{\ell^2} - \frac{\alpha_0 + \pi}{\alpha_0 - \pi} \frac{e_0 + \pi/2}{e_0 - \pi/2} \int_{\alpha_0}^{\alpha_0 + \pi} \int_{\epsilon_0}^{\epsilon_0 + \pi/2} (\vec{e}_3 \cdot \vec{e}_r) \frac{Z e f^2 \cos \epsilon \delta \alpha}{\ell^2} - \frac{\alpha_0 + \pi}{\alpha_0 - \pi} \frac{e_0 + \pi/2}{e_0 - \pi/2} \int_{\alpha_0}^{\alpha_0 + \pi} \int_{\epsilon_0}^{\epsilon_0 + \pi/2} (\vec{e}_3 \cdot \vec{e}_r) \frac{Z e f^2 \cos \epsilon \delta \alpha}{\ell^2} \]

where \( \vec{v} = (u, v, w) \), \( \vec{w}_t \) fall speed, \( \vec{e}_3 \) and \( \vec{e}_r \) unit vectors on vertical and radial directions.

**Simplification examples:**

- **No reflectivity weighting:**
  \( v_r({\vec{r}_0}) = \frac{\alpha_0 + \pi}{\alpha_0 - \pi} \frac{e_0 + \pi/2}{e_0 - \pi/2} \int_{\alpha_0}^{\alpha_0 + \pi} \int_{\epsilon_0}^{\epsilon_0 + \pi/2} (\vec{v} \cdot \vec{e}_r) \frac{Ze f^2 \cos \epsilon \delta \alpha}{\ell^2} \)

- **No hydrometeor fall speed:**
  \( v_r({\vec{r}_0}) = \frac{\alpha_0 + \pi}{\alpha_0 - \pi} \frac{e_0 + \pi/2}{e_0 - \pi/2} \int_{\alpha_0}^{\alpha_0 + \pi} \int_{\epsilon_0}^{\epsilon_0 + \pi/2} (\vec{v} \cdot \vec{e}_r) \frac{Ze f^2 \cos \epsilon \delta \alpha}{\ell^2} \)

- **No beam smoothing:**
  \( v_r({\vec{r}_0}) = \vec{v} \cdot \vec{e}_r - \vec{e}_3 \cdot \vec{e}_r \vec{w}_t \)
Approximations for fall speed $\bar{w}_t$

\[
\bar{w}_t := \begin{cases} 
\left( \frac{\rho_0}{\rho} \right) 0.5 \sum_{k \in S} \int_0^\infty \sigma b_k(D_k) \omega_{tk}(D_k) N^k(D_k) dD_k \\
\left( \frac{\rho_0}{\rho} \right) 0.5 \sum_{k \in S} \int_0^\infty \omega_{tk}(D_k) N^k(D_k) dD_k \\
\sum_{k \in S} \int_0^\infty N^k(D_k) dD_k
\end{cases}, \quad \text{if weighting by reflectivity;}
\]

\[
\bar{w}_t := \begin{cases} 
\sum_{k \in S} \int_0^\infty N^k(D_k) dD_k
\end{cases}, \quad \text{otherwise,}
\]

$\rho$ is air density, $\omega_{tk}$ is fall speed of a hydrometeor of species $k$, $k \in \{\text{cloud c, grapple g, ice i, rain r, snow s}\}$.

Take ice with generalized Gamma distribution as example to show the analytic formulation of numerators

**Weighting by reflectivity:** only Rayleigh scattering scheme implemented

\[
\bar{w}_{ti} = C_1 C_3 L^{1+\frac{bvr}{3}} \left[ \frac{\Gamma \left( \frac{2+\nu+bvr+1}{\mu} \right)}{\Gamma \left( \frac{\nu+1}{\mu} \right)} \right] \left[ \frac{\Gamma \left( \frac{\nu+1}{\mu} \right)}{\Gamma \left( \frac{\nu+2}{\mu} \right)} \right]^{2+\frac{bvr}{3}} + C_2 C_3 L^{1+\frac{bvi}{bgi}} \left[ \frac{\Gamma \left( \frac{2+\nu+bvi+1}{bgi} \right)}{\Gamma \left( \frac{\nu+1}{bgi} \right)} \right] \left[ \frac{\Gamma \left( \frac{\nu+1}{bgi} \right)}{\Gamma \left( \frac{\nu+2}{bgi} \right)} \right]^{2+\frac{bvi}{bgi}}
\]

**No reflectivity weighting:**

\[
\bar{w}_{ti} = f_m^\alpha a_{vr} \left( \frac{6x}{\pi \rho_w \rho} \right) \frac{bvr}{3} L^{1-\frac{bvr}{3}} \left[ \frac{\Gamma \left( \frac{\nu+bvr+1}{\mu} \right)}{\Gamma \left( \frac{\nu+1}{\mu} \right)} \right] \left[ \frac{\Gamma \left( \frac{\nu+1}{\mu} \right)}{\Gamma \left( \frac{\nu+2}{\mu} \right)} \right]^{\frac{bvr}{3}}
\]

\[
+ \left( 1 - f_m^\alpha \right) a_{vi} \left( \frac{1}{a_{gi}} \right) \frac{bvi}{bgi} L^{1-\frac{bvi}{3}} \left[ \frac{\Gamma \left( \frac{\nu+bvi+1}{bgi} \right)}{\Gamma \left( \frac{\nu+1}{bgi} \right)} \right] \left[ \frac{\Gamma \left( \frac{\nu+1}{bgi} \right)}{\Gamma \left( \frac{\nu+2}{bgi} \right)} \right]^{\frac{bvi}{bgi}}
\]
Test on fall speed $\overline{w}_t$
Efficient Radar Forward Operator for Operational Data Assimilation and Model verification within the COSMO-model

Figure 8: Dash boxes are optional. The configuration of the operator is namelist driven

We give our operator the name

"EMRADSCOPE" (Efficient Modular RADar Scanning forward OPERator)
Efficient Radar Forward Operator for Operational Data Assimilation and Model verification within the COSMO-model

Real convective case study

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Beam broadening/weighting</th>
<th>Beam bending</th>
<th>Scattering Schemes</th>
<th>Attenuation</th>
<th>Reflectivity weighting</th>
<th>Fall speed</th>
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<tbody>
<tr>
<td>E₀</td>
<td>No</td>
<td>43ERM</td>
<td>Rayleigh</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>E₁</td>
<td>No</td>
<td>SODE</td>
<td>Rayleigh</td>
<td>No</td>
<td>No</td>
<td>No</td>
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<tr>
<td>E₂</td>
<td>Yes</td>
<td>SODE</td>
<td>Rayleigh</td>
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<td>Yes</td>
<td>No</td>
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<tr>
<td>E₃</td>
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<td>Mie</td>
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<td>No</td>
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<tr>
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<td>Yes</td>
<td>SODE</td>
<td>Mie</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>E₅</td>
<td>Yes</td>
<td>SODE</td>
<td>Mie</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Figure 9 : Different configurations of the sensitivity experiments: successive upgrade to examine the effect of each configuration
Figure 10: Observed and simulated Doppler velocity (left) and reflectivity (right) PPI scans at elevation 2.5° on 30 June, 2012, 23:00, Türkheim.
Efficient Radar Forward Operator for Operational Data Assimilation and Model verification within the COSMO-model

Figure 11: Upper panel: PPI of Doppler velocity at different elevations in E5; Lower panel: Differences between E5 (with fall speed) and E4 (without fall speed)
Efficient Radar Forward Operator for Operational Data Assimilation and Model verification within the COSMO-model

Efficiency of EMRADSCOPE

<table>
<thead>
<tr>
<th>Experiment</th>
<th>EMRADSCOPE [s]</th>
<th>Total COSMO-model run [s]</th>
<th>Time increase [s]</th>
<th>Ratio [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$ (simple)</td>
<td>17.14</td>
<td>348.24</td>
<td>—</td>
<td>4.92</td>
</tr>
<tr>
<td>$E_1$ (+SODE)</td>
<td>24.16</td>
<td>354.65</td>
<td>6.41</td>
<td>6.81</td>
</tr>
<tr>
<td>$E_2$ (+pulse volume)</td>
<td>40.21</td>
<td>387.70</td>
<td>33.05</td>
<td>10.37</td>
</tr>
<tr>
<td>$E_3$ (+Mle)</td>
<td>62.28</td>
<td>419.71</td>
<td>32.01</td>
<td>14.84</td>
</tr>
<tr>
<td>$E_4$ (+attenuation)</td>
<td>63.81</td>
<td>426.59</td>
<td>6.88</td>
<td>14.96</td>
</tr>
<tr>
<td>$E_5$ (+fall speed)</td>
<td>73.78</td>
<td>461.48</td>
<td>34.89</td>
<td>15.99</td>
</tr>
<tr>
<td>$E_0^*$</td>
<td>204.19</td>
<td>6122.85</td>
<td>—</td>
<td>3.33</td>
</tr>
<tr>
<td>$E_4^*$</td>
<td>955.80</td>
<td>6688.86</td>
<td>566.01</td>
<td>14.30</td>
</tr>
<tr>
<td>$E_5^*$</td>
<td>1051.38</td>
<td>7090.19</td>
<td>401.33</td>
<td>14.83</td>
</tr>
</tbody>
</table>

Figure 12: Elapsed wall-clock time on NEC SX9: the first six experiments $E_0 - E_5$ are 4-hour model runs with one radar station; the last three experiments $E_0^*, E_4^*$ and $E_5^*$ (the same configuration as $E_0$, $E_4$ and $E_5$, respectively) are 26-hour runs with the whole radar network.
Case studies with all German radars

Animation of reflectivity PPIs at 1.5° in COSMO-DE domain: 4 September 2011, 12 - hour run, 16 radar stations (Rayleigh, offline propagation with 43ERM, no beam weighting),

Figure 13: PPI at 0.5° 15:00 24.06.2012: simulated reflectivity (left) and observed reflectivity (right), in overlap regions: values from lowest height
COSMO-DE model verification - Contoured Frequency by Altitude Diagrams (CFADS)

Figure 14: Simulated reflectivity (left) and observed reflectivity (right) PPI at 0.5°

- Lowest elevation, echoes mostly from the rain region
- Overestimated simulation probably because of the "unrealistic" model assumption about particle distribution function about rain

Figure 15: CFADs of simulated reflectivity (left) and observed reflectivity (right) for all elevations

- Too broad distribution in the model
- Reflectivity too high below melting level
- Reflectivity too low above melting level
Daily radar data monitoring

Figure 16: Scatterplot of simulated and observed Doppler velocity on 10 April, 2014, 00:00, Ummendorf, height in colorbar
Daily radar data monitoring: raw data
Daily radar data monitoring: quality flag evaluated
Daily radar data monitoring: dualPRF correction
Daily radar data monitoring: $\text{abs}(\text{obs} - \text{sim}) \leq 5 \text{ m/s}$
Superobservation

The amount of radar data must be reduced for the purpose of data assimilation $\implies$ superobservation technique

Goal: to achieve a relative homogeneous data distribution at least in horizontal direction
Superobservation

Step 1:
- construct a horizontal cartesian grid over the model domain

Cartesian grid
Superobservation

Step 2:
- start with the lowest PPI scan from one radar station
**Superobservation**

Step 3:
- loop over cartesian grid points, for each cartesian grid point find the radar bin on the PPI scan whose projection onto the horizontal plane is closest to this cartesian grid point.
Superobservation

Step 4:

- define the local area around the radar bin for superobbing
- calculate the average of values within the local area
- the average only accepted when variance smaller than some threshold value and the number of radar bins larger than some threshold value.
Superobservation

Step 5:
- do Step 3 - 4 for all cartesian grid points, i.e., find the closest radar bin, define the local area and calculate the average
Superobservation

Sixth Step:
- when the loop over all cartesian grid points is done, go to the next PPI scan
Superobservation

Example

**Figure 17**: PPI of Doppler velocity at elevation 2.5°. For superobbing, the cartesian grid spacing = 5.6 km, variance = 10 \( m^2/s^2 \), number = 3
Summary, publications and applications

Summary:
1. EMRADSCOPE has been developed, effects like beam bending and broadening, attenuation, fall speed are considered. Loop-up tables are coded for Mie scattering scheme
2. It is shown that the performance of EMRADSCOPE is promising and it is efficient for operational usage
3. Superobservation technique is implemented

Publications:
1. Book: Efficient Radar Forward Operator for Operational Data Assimilation within the COSMO-model (soon published, KIT Scientific Publishing)

Applications:
1. Model verification (D. Jerger, KIT)
2. Incorporated into daily radar data monitoring (K. Stephan, DWD)
3. Runs at MeteoSwiss in research mode (M. Betschart)
4. HD(CP)2 project: LES model verification strategies using radar, first with COSMO-model, later with ICON (D. Zhang, K. Wapler, DWD)
5. LETKF data assimilation experiments with Doppler velocity (Y. Zeng, DWD)
6. HERZ project: application in nowcasting and LETKF data assimilation (M. Würsch, H. Lange, LMU; T. Bick, Uni Bonn)
Now we can begin to assimilate radar data ...

Thank you for attention!